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ABSTRACT

The purpose of this study was to examine whether the mathematical structures inherent in grade 7 geometry curriculum objectives can be used to improve the communication of the objectives to students. Teacher inservice based upon geometrical properties and structures was combined with student teaching materials to try to improve student achievement in geometry. The subjects were 671 grade 7 students and 20 teachers in a large school board in Ontario (Canada). An analysis of the results reveal that the geometry objectives used in the study can be implemented successfully at grade 7 with the exception of rotational symmetry, some construction, and formal transformations. Achievement levels for students whose teachers received inservice in geometry were higher and there were no significant gender differences in achievement at grade 7 when an equal opportunity to learn was present. Included are: the introduction; a review of the literature; methodology; results; and conclusions, discussion, and recommendations. Appended are a list of geometry objectives, geometry evaluation and teacher instructions, student instructional materials, and teacher questionnaires. (KR)

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COMMUNICATION OF GEOMETRICAL STRUCTURE AND ITS
RELATIONSHIP TO STUDENT MATHEMATICAL ACHIEVEMENT

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Abstract

Communication of Geometrical Structure and its Relationship to Student Mathematical Achievement

The purpose of this study was to examine whether the mathematical structures inherent in grade 7 geometry curriculum objectives can be used to improve the communication of the objectives to students. Teacher inservice based upon geometrical properties and structures was combined with student teaching materials to try and improve student achievement in geometry.

The subjects in the study were 671 grade 7 students and 20 teachers in a large school board in the Province of Ontario. All of the teachers received specifically designed teaching materials which reflected the properties and structures from the content of geometry objectives in the board mathematics curriculum. One group of 10 teachers also received 3 sessions of inservice, one in each teaching term of the school year.

Student achievement from a September pre-test and a May post-test was analyzed and a p-value was obtained for each objective evaluated. Both the pre-test and the post-test contained 20 objective based geometry items, 6 measurement items and 10 numerical methods items.

Both teacher groups in the study completed a teacher questionnaire in the last term of the school year. The questionnaires were designed to gather data on teacher attitude and opinion about inservice and instructional materials in geometry.

Data analysis identified objectives with extremely high or low p-values. Paired t-tests allowed for comparisons between pre and post-test results for students of teachers from the inservice and non-inservice groups. Gender differences were also examined using results from both tests. Teacher

questionnaires yielded responses and comments all of which were included in the study.

The analysis revealed that the geometry objectives used in the study can be implemented successfully at the grade 7 level with the exception of rotational symmetry, some constructions and formal transformations. Achievement levels for students whose teachers received inservice in geometry were significantly higher but it is not clear whether the emphasis on structures and properties created the difference.

There are no significant gender differences in achievement at grade 7 when an equal opportunity to learn is present. Also hands-on instructional materials are supported by teachers but require inservice for clarification.

The results obtained in this research have implications for the teaching of geometry in the elementary school. Well sequenced objectives, teacher inservice and hands-on materials are all supported.

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Chapter 1

Introduction

Freudenthal (1973) described geometry as "the experience with, and interpretation of, the space in which a child lives, breathes, and moves." This important statement about elementary school children and their environment paralleled recommendations by Bonwell, Saunders and Tehta (cited in Bishop, 1983) outlining teaching approaches for geometry that involved experimenting with materials and representations as well as classifying, defining and analyzing why certain relationships occur.

There seemed to be a consensus within the global educational community that content and processes related to geometry should be included in the elementary school mathematics curriculum. Support for geometry in the curriculum arose as early as 1959 at a conference at Wood Hole, Massachusetts which emphasized teaching processes and a conference in Cambridge, Massachusetts in 1963 which emphasized content in geometry (Resnick and Ford, 1981). In Ontario, a report on the teaching of geometry was published as early as 1967.

The Report of the Ontario K-13 Geometry Committee provided a rationale for the inclusion of geometry in the curriculum in terms of the following points:

- a. Geometry is the natural language of spatial concepts and relationships.
- b. Geometry plays a basic role in science engineering and other professions.

- c. Geometry has a strong aesthetic connection.
- d. Geometry can give aesthetic and intellectual pleasure and stimulation.

Recent developments in mathematics education in Canada and the United States continue to support the teaching of geometry in the elementary and secondary school curriculum. The 1985 curriculum guideline for mathematics in Ontario schools lists geometry content in every grade level of the Intermediate and Senior Divisions beginning at grade 7. Most school boards in the Province of Ontario are now implementing these curriculum directions. At grade levels below the Intermediate Division, strong geometry components have also been included in Ontario Assessment Instrument Pools and Provincial Review materials.

Educators in the United States also lend their support to the teaching of geometry in schools. A recent international symposium at Syracuse University reviewed teaching sequences for geometry content at all levels and discussed relevant developments in Van Hiele's levels of learning geometry.

At the same time the National Council of Teachers of Mathematics (1987) has produced a set of curriculum standards for school mathematics which includes statements about geometry content and the processes of teaching geometry. Three sections of the standards document (K-4, 5-8, 9-12) emphasize directions for the teaching of geometry in schools.

Further evidence of the significance of geometry in the school curriculum in the international community is available in the Second International Mathematics Study (SIMS). Robitaille and Travers (1986) used data gathered from 20 countries collected by the International Association for the Evaluation of Educational Achievement (IEA) during the time period 1980 to 1982. Writing for a United Nations publication, these authors described a population of international 13-year-olds whose achievement was measured on an item pool of

mathematics questions, where 157 items mirrored geometry content. It is noteworthy that the content of the geometry items was agreed upon by all countries.

It is clear that educators agree on the importance of geometry in the school curriculum. However, it is not clear whether the agreement extends to the content domain or sequence. For a specific grade level the teaching objectives and teaching sequence in geometry might differ from environment to environment and from teacher to teacher. The purpose of the present study is to examine this issue as well as measuring the achievement of students relative to geometry content.

1.1. The Problem for the Study

Robert Morris (1986) reviewed current practices of teaching geometry in several countries and concluded in his preface: "There is no consensus on the content of the school geometry curriculum." This precise and somewhat conclusive statement was followed at the same time in the same publication by the writing of two other educators who indicated that "no one seems to be sure why geometry is there, how much time to devote or how best to teach it" (Robitaille & Travers, 1986, p. 29).

Bishop (1983) had already stated the problem of consensus on content in a more generalized format. He did not disagree with the inclusion of geometry in the curriculum. However, he saw "no reason to expect a consensus on what constitutes geometry education" (p. 175). Coxford (1978) had also supported the lack of consensus on sequence of geometric material in elementary school mathematics materials. In Coxford's words "there appears to be few universally accepted guidelines" (Coxford, 1978, p. 324).

This study involves geometry in the elementary school curriculum and in particular geometry for adolescents at the grade 7 level in the Ontario, Canadian system of education. The 'consensus of content' problem will be resolved by

supporting current teaching objectives contained within a large school board Mathematics Core Curriculum. This will be done from a chronological and mathematics education perspective with Canadian as well as International reviews being included.

Once an agreed upon content sequence has been established, the focus of the problem shifts to the teaching of geometry. What kind of relevant research on the teaching of geometry could be done?

In a review of research directions in geometry, Coxford (1978) indicted that "we need to begin to relate geometry understanding to the type of geometric experiences learners experience" (p. 323). Geometric understandings relative to the nature of the instruction used by teachers was also cited in the category of needed research. Coxford was making it clear that the learning of geometry depended upon the learner's experiences and upon formally organized and planned instruction.

It is the thesis of this study that analyzing the content of grade 7 geometry objectives for mathematical structures, by teaching teachers about these structures and by teaching students using structures, achievement in geometry may increase. This research study will attempt to determine whether the mathematical structures inherent in grade 7 geometry curriculum objectives can be used to improve the communication of the objectives to students. Will in-service teacher training on geometric structures related to specific teaching objectives bring about an improvement in the achievement of students?

1.2. Notes on Structure

To illuminate the specific nature of this study it becomes necessary to discuss 'structure' as it applies to the learning of mathematics and how 'structure' relates to the geometry objectives outlined above.

Lesh, Landau and Hamilton (1983) verbalized the importance of structure in mathematics education as follows:

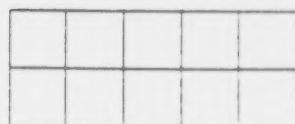
To most mathematicians, mathematics is the study of structure, the content of mathematics consists of structures and to do mathematics is to create and manipulate structures (p. 268).

As early as the 1950's mathematicians felt that children could appreciate intuitively the concepts and relationships that form the structure of mathematics. Resnick and Ford (1981) indicated that "central to the goals of Wood's Hole, 1959 (process) and Cambridge, 1963 (content) was the teaching of mathematical structures" (p. 127). These authors supported a structure oriented approach as a worthy goal for mathematics instruction. This implied a "design of teaching that presents the underlying structures of mathematics elegantly and simply while at the same time taking into account the cognitive capabilities of learners" (p. 127).

In his discussions about observations of children Piaget (1968) defined structure. Van Hiele (1986) quotes Piaget as saying, "structure has totality, structure is achieved by transformations and structure is autoregulating" (p. 27). Later, Glaserfeld (1987) and Mason (1987) linked structure to children and teachers. Glaserfeld indicated that children link problems with solutions by "the awareness of the structures and operations" (p. 16). Mason observed that teachers of mathematics "have a propensity to concentrate on naming mathematical things" (p. 77) and this is "an example of the attraction of surface structure over deep structure" (p. 77). Lester (1983) had already outlined the importance of structure when he noted that an attribute of a good problem solver was the ability to "quickly and accurately see the mathematical structures" (p. 235). Silver's (1981) problem solving research supported Lester's claim.

Structure has been seen as important in the process of learning/teaching of mathematics. Now it must be linked specifically to the geometry in this grade 7 study.

The decision to emphasize structure in geometry came from discussions about structure initiated by Van Hiele. Van Hiele (1986) stated that "everything that has something to say has it because of a structure." His examples (Van Hiele, 1986, p. 14) of geometric structures included these:



At an international symposium on the teaching of geometry Van Hiele indicated that, "most adolescents would be located in level one or level two of the following levels of thinking hierarchy":

- Level 1 - visual
- Level 2 - descriptive
- Level 3 - theoretical
- Level 4 - formal
- Level 5 - logical

Van Hiele explains the visual level by discussing "communication about structures that can be observed." This is followed by the descriptive level which implies "thinking about the nature of structures" (Van Hiele, 1986, p. 83).

Hoffer (1983) outlined Van Hiele's level one under the title of 'directed orientation'. Here he saw "carefully sequenced activities for student exploration so that they will become familiar with the characteristic structures." Hoffer continued by labelling level two as 'expliciting' and viewed children as "building from previous experiences and expressing opinions about the inherent structures" (Hoffer, 1983, p. 208).

The recognition of the importance of structure and the acceptance of the fact that levels of thinking (Van Hiele) can be identified is essential for this study. Most grade 7 students would be at the visual level and/or the descriptive

level. It follows naturally that alerting teachers to structures should improve student achievement related to specific teaching objectives. In particular the geometrical properties are the structures which will be outlined for teachers at in-service presentations and outlined for students in specifically designed instructional materials utilized with this study.

Chapter 2

Review of the Literature

The problem for this study as outlined in the introduction centres upon identifying structures/properties contained within geometry content specified in objectives for grade 7 students. Structures are outlined for students in specifically designed instructional materials. In the literature review attention will be given to the teaching of geometry, particularly, at the elementary school level. Other factors to be given consideration include: geometric properties as mathematical structures, teacher inservice relative to student achievement, levels of learning in geometry and gender differences in student achievement on geometry content.

2.1. Research on Pre-deductive Geometry

As he reflected on geometry in elementary education Bishop (1983) stated that, "we do not have an extensive or comprehensive corpus of research from which they can draw ideas in tackling the issues surrounding the teaching of geometry" (p. 176). Bishop was referring to mathematics educators and supported his thesis by illustrating that the Journal for Research in Mathematics in 1979 and 1980 carried 376 entries; only 18 entries concerned space and geometry in some way. The literature search which was done for this study would also support the fact that little research has been done relative to geometry or the teaching of geometry particularly in the elementary schools. Research is needed and this supports the present study.

By 1960, an intuitive and pre-deductive geometry became acceptable in

elementary school mathematics programs. Pre-deductive geometry teaching objectives were listed in the Cambridge Conference Report (1963), Geometry of the Primary Grades (1960) and the School Mathematics Study Group Project (1961). Fehr (1960) defined predeductive geometry as a physical representation of the world in which we live. He also indicated a general acceptance for this kind of geometry to "play a larger role in elementary education."

Andrews (1966) analyzed the role of geometry at the pre-deductive level in school mathematics. From his historical review, Andrews concluded that our "concern must be with objectives, content and presentation" (p. 101). After analyzing six elementary mathematics programs Andrews listed a series of questions for program analysis.

Andrews (1966) also concluded that, "authors cannot assume that the teacher has an adequate background in geometry. Background information must be provided; there is also a need for suggested methods of presentation of content that is very different from traditional content" (p. 3).

In 1969, Grimes continued to lament the "little or no emphasis on geometry" (p. 1) in elementary school programs. He noted that the "elementary school mathematics teacher is seriously handicapped without some formal preparation in the area of geometry" (p. 1). His study outlined nine chapters of modern geometric development for elementary school mathematics teachers.

Grimes (1969) and Cathcart (1975) both cite the Committee on the Undergraduate Program in Mathematics (CUPM) as a source of content for geometry programs for teachers. CUPM (1961) included the following guidelines:

A study of space, plane, and line as a set of points, considering separation properties and simple closed curves; the triangle, rectangle, circle, sphere, and the other figures in the plane (p. 990).

Ten years later, in 1971, CUPM published an up-dated set of recommendations which included modification and expansion of the informal geometry to

incorporate considerable treatment of transformational geometry and coordinate geometry.

In 1975, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education noted a "continuing controversy over the content and organization of high school geometry" and "evidence of fundamental disagreement over the purpose of school geometry as a whole" (p. 15). Bishop (1983) also commented on a lack of consensus as to what constitutes geometry education. This discussion on consensus carries through to 1986, when Robitaille and Travers used data from the Second International Mathematics Study to justify an "impression of a lack of clear direction." Referring to geometry for 13-year-olds these researchers indicated that "no one seems to be sure why geometry is there, how much time to devote to it or how best to teach it" (Robitaille & Travers, 1986, p. 29).

While the literature does not support a consensus on content there appeared in the late 1960's and early 1970's a strong support for active student learning. Bring (1971) used a unit on metric and non-metric geometry to see if amount and type of concrete activities affected learning. He concluded that information such as age, sex, ethnic background, and reading levels could be combined to predict achievement in geometry. Prediction of achievement was highly related to student performance on the standardized Metropolitan Achievement Test and the inclusion of concrete activities did increase achievement. Biggs and MacLean (1969) had emphasized activity and concrete materials and they believed that the only way for prospective teachers to understand the potential of active learning is to experience it themselves.

Wardrop (1970) studied the active use of geometry enrichment exercises and their effect on teacher attitudes. He concluded that "enrichment exercises of a geometric nature do not affect the attitude or achievement of prospective elementary teachers" (p. 15). However, the emphasis on active learning

continued. Kleinhaus (1976) developed materials of a mathematics laboratory approach which included the CUPM geometry recommendations. She documented growth in student achievement and concluded that the laboratory/activity approach was sound. Wong (1972) had also concluded "it is essential that, for the teachers as well as the student, geometric content be acquired through an informal, inductive manipulative approach" (p. 91).

2.2. Geometry and Structure

The literature which is based on learning geometry content or the structures of mathematics spans three decades which begin in the middle 1960's after attention was drawn to the teaching of geometry. In 1965, Fehr stated that geometry instruction presented the most troublesome problem. Andrews (1966) in a study which analyzed the role of pre-deductive geometry in school mathematics, commented on the need to determine geometry objectives and teacher training.

By the next decade Piaget (1971) was describing mental and mathematical structures. For him a symbol like $\square$ could be used to convey (a) 4 corners - each equal, and each 90° ; (b) 4 straight sides, with opposite sides parallel. Piaget indicated that these are structures represented by a multifaceted symbol or image.

Cathcart (1975) in a study relating activity instruction and teacher training highlighted a recommendation from CUPM which suggested that mathematics should be taught in ways that foster active student involvement in the development and presentation of mathematical ideas. Wirszup (1976) in a review of Russian psychological studies pointed to the fact that "a network of relations which is not founded on previous experience risks, as we all know, being forgotten in a short time" (p. 81). Lesh and Mierkiewicz (1978) suggested that a student's perceptions were also a critical factor. They defined perception as the senses gathering information from the environment. These researchers indicate

that "there is a great deal of ambiguity about similarities and differences between perceptual and conceptual processes" (p. 7). The ambiguity applies at the instructional and developmental phases of geometric concepts. Martin (1976) also indicated that perception was important to help youngsters develop representational systems that will permit geometric objects to be perceived as we want them to be perceived.

Students will only be able to perceptually process a limited number of items at one time. Lesh (1978) indicated that for efficiency each structural item should carry as much information as possible and that it would be economical to store only typical characteristics. Research on perceptual and conceptual processes is also required. Lesh (1978) also indicated that "we need baseline data on what children see." Do they see the structures of geometry and do we adequately define the structures for each student? (cited in Coxford, 1978, p. 323). The characteristics of the geometry content determines student behaviour.

Any representation must emphasize certain properties and "children will distort what they 'see' in order to represent what they understand" (Coxford, 1978, p. 323).

We need baseline data on what children 'see' as an adult sees (Coxford, 1978). Coxford also stated in his review of necessary research that "we need to help youngsters develop representational systems that will permit geometric objects to be perceived as we wish them to be perceived" (p. 324). Do they see the structures of geometry and do we adequately define the structures for each student?

Charles (1980) in one of the few geometry studies listed in the *Journal of Research in Mathematics* cited Rosenshine and Furst (1971) as identifying that "teacher clarity was a consistent correlate with high pupil achievement" (p. 11). In his study, Charles (1980) used two training sessions to familiarize teachers

with the content of instruction. Exemplification (E) moves served the purpose of illustrating the relevant and irrelevant attributes of a concept. Characterization (C) moves were statements about relevant or irrelevant attributes. E and C moves related to bilateral and rotational symmetry facilitated the acquisition of these concepts by elementary pupils. The study recommended "further attention for research on teaching strategies in mathematics" (Charles, 1980, p. 19).

Teaching the structures of mathematics was highlighted by Resnick and Ford (1981). They pointed out that the goals of geometry conferences such as Cambridge (1963) and Wood's Hole (1959) contained issues on teaching of mathematical structures. For these authors the understanding of the structures of mathematics implied: 1) interrelation; 2) rules of mathematics. Also, they recommended a design of teaching that presents the underlying structures of mathematics elegantly and simply while at the same time taking into account the cognitive capabilities of the learner.

In 1983, Hoffer suggested that carefully sequenced activities for student exploration will lead to familiarization with characteristic structures. He said that students should build from previous experiences and express opinions about the inherent structures. Bishop (1983) supported the views of Hoffer when he noted concerns with helping pupils gain knowledge and skills in the mathematical interpretations of space. Van Hiele (1986) stressed that "everything that has something to say has it because of a structure" (p. 11). His research defined a visual level of learning that was "communication about structures that can be observed." The descriptive level was "thinking about the nature of structures" (p. 83). Crowley (1987) outlined teaching experiences based on Van Hiele structures and Farrell (1987) published strategies and activities based on geometrical structures which had been used in inservice for secondary school teachers.

Skemp (1987) reviewed structure from a psychology of learning perspective. For Skemp "concepts can be interrelated and result in structures" (p. 181). "The

study of structures themselves is an important part of mathematics and the study of the ways in which they are built up and function is at the very core of the psychology of learning mathematics."

The National Council of Teachers of Mathematics (1987) recognized the importance of structure in geometry. They emphasized that "understanding the properties of and relationships between geometric figures is essential to applying these ideas to the development of other mathematical concepts." Schoenfeld (1988) in a study involving high achieving grade 10 students found that the students were able to do well on standardized achievement tests but learned very little about the structure of the mathematics in their geometry course. Schoenfeld found that the form of an answer was more important than the mathematics being discussed. Good and Grouws (1987) had noted similar results for arithmetic and concluded that most developmental lessons focused on memorization rather than on understanding and visualizing concepts.

McDonald (1989) concluded that "formal operational students structured subject matter content significantly more like subject matter experts than concrete operational students" (p. 76). Although her geometry study utilized secondary students as subjects she did focus on issues of structure that are also important for elementary students. She cites (Johnson, 1969; Shavelson, 1974; and Thro, 1978) as examples of studies on processing information that indicated that students differentiate by shape, type of figure and construction of "class inclusion principles (all triangles are polygons)" (p. 76).

From the evidence in the literature right up to McDonald (1989) it is clear that a distinction must be "made between content structure and cognitive structure." Cognitive structure is a hypothetical construct representing a person's organization of concepts as long-term memory (McDonald, 1989). For the purposes of the present study content structure is the properties of the geometry content that is taught (all polyhedra have a certain number of straight edges, vertices and faces).

2.3. Levels of Learning in Geometry

The problem in this study deals with instruction and learning in geometry. Levels of learning have become an important issue in the literature about teaching geometry.

As early as 1967, the Ontario Report on Geometry noted that "the psychological and pedagogical defects now recognized in classical geometry courses are partially responsible for this low level of geometric literacy." The report concluded that "on most occasions the work should not exceed the Piaget stage of the children involved." For students above 12 years, grade 7 and upward the Ontario report includes a stage of formal operations as follows:

observations, constructions, generalization of particular to general cases, and enunciation of general properties (Duff, 1967, p. 9).

While the Ontario Report concentrated on Piagetian levels, Fuys (1985) pointed out that between 1960 and 1964 the Russians had substantially revised their geometry program to take into account the principles of Van Hiele. Wirszup (1976) in a review of Russian psychological studies brought the work of Van Hiele to the attention of North American educators. He concluded that all development proceeds basically under the influence of learning and therefore depends on the content and methods of study.

Coxford (1978) discussed levels of learning and instruction. He briefly described the Russian levels as: (a) information; (b) directed orientation; (c) explanation; (d) free orientation and (e) integration. In the discussion it was noted that mathematical learning depends upon formally organized and planned instruction and that level I activities would be focused on determining the properties of individual figures.

In 1983, Mayberry was even more specific. She recommended that level I activities should involve drawing, measuring and other activities with concrete representations of geometric figures. In reference to instruction in geometry she

indicated that if the language of instruction is at a higher level than a student's thought processes, the student will not understand the instruction. Fuys (1985) in a study based on clinical interviews with grade 6 students concluded that the entry level of these students was Level 0 and the potential levels after instruction were I or II. In 1988, Fuys extended his observations by stating that ninth graders' entry levels were mainly level I. Fuys (1988) also noted that progress from one level to the next is more dependent upon instruction than one on age or biological maturation.

Burger and Shaughnessy (1986) studied students in grade 1 through 12 using the same definition for level 0 and 1 that Fuys (1985) has described:

Level 0 (Visualization) - The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components.

Level I (Analysis) - The student reasons about geometric concepts by means of an informal analysis of compound parts and attributes. Necessary properties of the concept are established (Burger & Shaughnessy, 1986, p. 31).

Using these levels, Burger and Shaughnessy (1986) concluded that "the students' behaviour on the tasks was consistent with the Van Hiele's original general description of the levels" (p. 31).

The literature is consistent in linking levels of development in geometry with instruction. This supports the problem posed in the present study where teachers use instructional materials to transmit geometrical structures that are contained in the visual and descriptive geometry levels.

2.4. Mathematics Inservice

Inservice has been consistently viewed as a means of changing instruction in mathematics. However, the results of inservice are somewhat inconsistent.

Ruddell and Brown (1964) did an experimental study with three groups of

teachers to measure the effectiveness of inservice. Group A received highly concentrated inservice presented by trained consultants. Group B had ten one-hour discussion meetings and group C received no information. These researchers reported significant changes in teacher knowledge for groups A and B but no changes in student achievement for any group.

Ebel (1969) reminded us that teachers must receive training to remain knowledgeable in this age of increasing technology and changing methods. Also in 1969 Norris reported a study with 18 grade 6 teachers. After a 3-hour college course his findings were inconclusive concerning the relationship between teacher knowledge and student achievement. Wong (1972) linked students and teachers in a study of geometry through inductive exercises. He concluded that it is essential that, for the teacher as well as the student, geometric content be acquired through an informal, inductive manipulative approach.

Anderson (1978) in a study relating inservice and computer technology reported a positive effect on student achievement on problem solving. However, Shulman (1979) a year later again brought the focus back to changes in teacher behaviour. He claimed that "any changes in curriculum and instruction must be mediated through the minds, motives and activities of teachers" (p. 77). Haley (1980) after conducting inservice with teachers concluded that there was a significant difference in student achievement as measured by the Stanford Achievement Test-Mathematics Battery. Berman (1981) used the achievement of elementary students to measure the effect of inservice on teachers. Teachers were given 24 hours of inservice over a period of 3 months. The inservice was based on an activity approach to teaching metric measurement. Berman reported significant gains in knowledge for both teachers and students.

Cuevas (1983) claimed that the research findings are not clear as to the benefits of inservice in terms of student learning. He noted that, "most inservice is aimed at improvement of the teachers' classroom behaviour" (p. 22). In his

study inservice made little or no difference on mathematics achievement and he concluded that transfer of skills may not occur as a result of inservice. McMahon (1983) was cited by Morrison and Brady (1985) as indicating that, inservice education often has not been considered as an effective means of upgrading.

Morrison and Brady (1985) report that there have been few changes in traditional inservice models and their solution for effective inservice is a planned systematic process. Shaughnessy (1987) was more precise when he said, "it is important for those of us doing inservice projects to attend to the mathematics content we wish to model" (p. 159).

Good and Grouws (1987) recognized that, "there are many critical weaknesses in the way teachers teach mathematics" (p. 778). They designed an inservice model to change the teachers' view of mathematics (production of correct answers) to teachers' developmentally "facilitating the meaningfulness of an idea by learners." The steps in development are:

1. Attending to prerequisites.
2. Attending to relationships.
3. Attending to representation.
4. Generality of concepts.
5. Attending to the language of mathematics (Good & Grouws, 1987, p. 780).

In their original model, Good and Grouws (1987) the teachers received two brief inservice sessions which conveyed information about key concepts. A teacher manual was then distributed. Later the inservice was extended to "give attention to mathematics content, methods of teaching mathematics and management issues" (p. 781). The "research evidence continues to accumulate that an informed implementation of this framework results in increased student learning" (p. 781).

The inservice learning in this study combined with developmental instructional materials attempts to build upon the characteristics of successful inservice described in the literature outlined above. Modelling of mathematics structures enhanced by developmentally oriented instruction should improve student achievement.

2.5. Gender Differences in Mathematical Achievement

In the present study there will be an opportunity to compare the population that is female with the population that is male. Tobias (1978) highlighted female differences in mathematical achievement in her well accepted outline of 'math anxiety'. She concluded that it was acceptable for female students not to do well in mathematics. Skypeck (1980) confirmed this viewpoint when she said, "boys just try harder, whereas girls find it permissible to believe that they are not supposed to be smart in mathematics" (p. 27).

Brush (1980) noted that throughout elementary school, the sexes rarely differ in their attitudes, but after the age of 12 any sex differences which are found show that girls dislike mathematics more than boys. Fennema (1979) in the first of a series of studies found that at about the 7th grade differences on higher level cognitive tasks are in favour of males.

Fennema (1979) also noted that for 12 countries represented in the 1964 International Education Achievement study males surpassed females. In a 1981 study, Fennema concluded that when both females and males study the same amount of mathematics, differences in learning mathematics are minimal. By 1985, Fennema had summarized theoretical models explaining sex-related differences in mathematics. The summary included factors such as curriculum, environment and participation.

Hanna (1987) addressed the issue of gender differences in geometry on a paper presented at Syracuse University. She analyzed the data from 20 countries

that participated in The Second International Mathematics Study. Data analyzed indicated that differences in performance are never in the girls' favour. However, in 10 of the 20 countries there were no significant differences between the sexes in achievement. These results on 51 geometry items seem to indicate that girls are capable of achieving as well as boys if the geometry is taught. In a second paper, Hanna (1988) observed that there were overall differences in achievement in geometry and measurement. She categorized geometry and measurement as being least taught and concluded that girls and boys assimilate the subject matter taught in class equally well. Differences in geometry achievement may be due to out-of-class experiences and to psychosocial processes rather than to biological differences. This study entails the teaching of the same geometry objectives by the same teachers, using the same instructional materials to both male and female students. If the conclusion drawn by Hanna (1988) is correct then the results of the present study should show no difference between male and female populations.

2.6. Content and Teaching Objectives

Prior to the publication of the 1967 Geometry Committee report in Ontario, projects such as Geometry of the Primary Grades (1960) at Stanford University and School Mathematics Study Group (1961) at Yale University were recommending pre-deductive geometry programs for the elementary school. In the summer of 1963, the results of a conference at Cambridge, Massachusetts also recommended geometry for elementary schools.

In the period from January to July of 1966 a committee to conduct an integrated study of geometry and the teaching of geometry in Ontario schools met and prepared a report. This committee included world-renowned mathematicians such as H.S.M. Coxeter and W.W. Sawyer as well as representatives from the schools in the Province. The committee report was published by 1967. Detailed

proposals for the geometry curriculum were presented in three stages. The three stages corresponded to primary grades K-6(a), intermediate grades 7-10(B), and senior grades 11-13(c). Each stage was then subdivided into units corresponding to single grades. Stage B1 matched the grade 7 level which is the focus of this study. Figure 2-1 outlines the content listed for Stage B1 in the 1967 report.

Figure 2-1: Stage B1: Ontario Geometry Objectives

a. Rectilinear figures:

- classification (by number of sides) and construction of regular (and irregular) polygons from written descriptions using compass, protractor, t-square.
- classification of various quadrilaterals - parallelograms, rhombuses, squares (Venn diagrams can be used to link algebra and geometry).

b. Triangles:

- paper folding could be used to illustrate the concurrence of angle bisectors, altitudes, right bisectors, medians; sum of three angles.
- plane mirrors (reflections) could be used to discuss the symmetries of isosceles and equilateral triangles.

c. Mensuration:

- perimeter and area of simple figures (rectangle, square, triangle, parallelogram, trapezium).
- area of figures that can be decomposed into triangles (e.g. hexagons).

The 1985 curriculum guideline for mathematics in the Province of Ontario continues to reflect the thinking of the 1967 geometry committee. Geometry in this guideline was listed under the three categories of basic notions in geometry, three-dimensional geometry and transformations. In the notations for basic notions in geometry the Ministry points out that "emphasis should be on clarifying the classifications identified in earlier grades, making specific the

geometric properties related to the classifications, and using correct terminology" (p. 32). For three-dimensional geometry the emphasis on actual objects and in transformations students should have an opportunity to develop visual perception and reinforce the identification of properties" (p. 33). Clearly the properties and structures of geometry are considered as important.

Relative amounts of geometry in the intended curriculum for Ontario 13-year-olds were presented by Robitaille and Travers (1986) in Table 2-1.

These authors concluded that "at the level of the intended curriculum Ontario has considerably more geometry content in its curriculum" (p. 25).

A year later in 1987 the National Council of Teachers of Mathematics had come to recognize geometry as an important part of the standards for mathematics curriculum in grades 5 to 8.

The NCTM statement reads as follows:

In grades 5-8, the mathematics curriculum should include the exploration of geometry in a variety of situations so that students can:

- identify, describe and compare geometric figures.
- visualize and represent one, two, and three dimensional geometric shapes.
- understand geometric relationships and their consequences.
- apply geometric properties and relations in solving problems and in developing other mathematical concepts.
- recognize and appreciate geometry as a means of describing the physical world.

In accordance with the renewed emphasis on geometry for the elementary school and to match the Ontario Ministry of Education guidelines, The Peel Board of Education in 1987 included geometry objectives in all levels of the Mathematics, Core Curriculum. Figure 2-2 represents three school terms of geometry objectives for grade 7. These were used as a base for this study. The objectives reflect the philosophy and guidelines outlined above.

Table 2-1: Indices of Curriculum Coverage in Geometry

Topic	Number	British Columbia	Ontario	United States
Classification of plane figures	6	0.83	1.00	0.83
Properties of plane figures	8	0.63	1.00	0.75
Coordinates	7	0.43	0.71	0.57
Congruence of plane figures	4	0.50	1.00	0.75
Simple deductions	4	0.50	1.00	0.25
Spatial visualization	3	0.67	1.00	1.00
Similarity of plane figures	6	0.33	1.00	0.83
Informal transformations	4	0.00	1.00	0.00
Pythagorean triangles	3	0.67	1.00	0.33
Formal transformations	4	0.25	0.25	0.00

Figure 2-2: Grade 7 Geometry Objectives

Term 1 - September, October, November

1. Classify and identify polyhedrons.
2. Identify relationships of edges and faces in polyhedrons.
3. Construct regular polyhedrons from nets.
4. Sketches of 3-D objects.
5. Classify angles by size.
6. Properties of intersecting, perpendicular and parallel lines.
7. Classify triangles by sides, angles and symmetry.
8. Classify quadrilaterals by sides, angles and symmetry.
9. Identify congruent figures and their properties.
10. Line symmetry in plane figures.
11. Rotational symmetry in plane figures.

Term 2 - December, January, February

12. Construct geometry figures.
13. Symmetry and congruence in designs.
14. Recognize and draw lines of symmetry.
15. Graph ordered pairs.

Term 3 - March, April, May

16. Slides, flips, and turns.
17. Create tiling patterns.
18. Properties of tiling patterns.
19. Basic properties of translation, reflection and rotation.
20. Draw images under translation, reflection and rotation.
21. Solve tiling puzzles.
22. Enlargements and reductions.

The objectives listed in Figure 2-2 reflect the Ontario view of geometry which as Robitaille and Travers (1986) indicate is considerably more content than other IEA countries. There is not a clear international consensus on inclusion of geometry content in the curriculum but these objectives (Figure 2-2) do represent a consensus on content for The Peel Board of Education and for the purposes of this study.

2.7. Summary

Literature reviewed for this study provide evidence that geometry is becoming increasingly important particularly at the pre-deductive levels. However, notwithstanding the consensus on teaching geometry in the elementary school there remains some difficulty in getting agreement on curriculum content for geometry. Jurisdictions at the Provincial and Board levels have agreed on specific objectives for various ages of students but International agreement has yet to be achieved.

More research related to instruction and development of geometrical processes is required. Researchers to-date agree that students and teachers should both learn mathematics and in particular geometry by actively becoming involved with mathematical concepts.

Structures and properties related to geometry need to be modelled for students and teachers. The concepts have to be presented with emphasis on developmental processes and on instructional materials which match appropriate developmental levels in the students.

Studies done with students and preservice teachers conclude that the Van Hiele levels for teaching geometry are appropriate. The first two levels visualization and analysis which emphasize perceived properties and compound parts and attributes respectfully are appropriate teaching levels for grade 7 students.

Inservice studies lead to a very clear direction. You can change the levels of achievement and knowledge in both students and teachers by carefully designed inservice. The keys to success appear to be active involvement, careful delineation of mathematical content, modelling of structures and properties and facilitating developmental processes for meaningful acquisition. Gender differences in geometry achievement are difficult to relate to inservice or

instruction. Large scale assessment results (SIMS) do support higher achievement for males but when opportunity to learn is accounted for girls achieve as well on content taught. The difficulty with geometry is that it is least taught. Sex related achievement differences may be more psychosocially than biologically determined.

The majority of literature reviewed in the categories of geometry in the curriculum, the structure of mathematics, levels of learning for geometry, mathematics inservice and gender differences in mathematical achievement supports the theses of the present study. By changing teachers' perceptions of geometry and providing appropriate instructional materials positive increases in student achievement should occur.

Chapter 3

Methodology

The purpose of this chapter is to describe the population and sample of the study. This will be followed by procedures, geometry objectives, student evaluation instruments, teacher inservice, and teacher questionnaires.

3.1. Population and Sample

This study was a single element included in an extensive program review of elementary and secondary school mathematics within a large board of education, in southern Ontario. At the time of the study (school-year 1988-89) The Peel Board of Education was the largest public school board in the Province of Ontario with 134 elementary schools and 30 secondary schools with a student enrolment of over 86,000 students.

The school board maintains a Program Department as a consultative service available to all schools. This Department includes 16 Coordinators who relate to specific academic subject disciplines. A Coordinator of Mathematics assumes the responsibility for mathematics programs in both elementary and secondary schools. The Coordinator of Mathematics solicited the assistance of the boards' Chief Research Officer to design and implement a system-wide review of mathematics programs at all grade levels. Included in this review was the geometry study outlined here. This geometry component was designed and implemented by the Coordinator of Mathematics who is the author of the present study.

In December 1987, the decision to work solely with geometry at the grade 7

level was made. A letter soliciting teacher volunteers was distributed to Principals of all schools with grade 7 students. This letter was accompanied by an initial description of the proposed geometry study for the information of teachers who wished to volunteer (see Appendices A and B for samples of the letter and teacher information sheet).

By May, 1988 an outline of geometry objectives for the school year 1988-89 was delivered to 25 teacher volunteers in 17 different schools (see Appendix C). When the study actually started in September 1988, the number of teachers involved was twenty. The number of grade 7 classes in the study numbered twenty-five and this included 671 students.

3.2. Procedures

As the names of teacher volunteers were submitted to the Coordinator of Mathematics they were assigned to an inservice group or a non-inservice group. If a school had more than one volunteer the teachers were assigned to different groups wherever possible.

The teachers who were designated 'Inservice' would receive inservice as well as instructional materials and a second group of teachers were designated 'Materials Only' and would not participate in the inservice. Each of the ten teachers in the first group were assigned code numbers from 101 to 110 inclusive and the second group was 201 to 210 inclusive. The study groups with numbers of students are shown in Table 3-1.

3.3. Geometry Objectives

For the purpose of directing the instruction for students 22 teaching objectives were defined for geometry in the Board's Mathematics Core Curriculum. All of the teachers were asked to organize the teaching of geometry by subdividing the 1988-89 school year into three terms and specific geometry objectives were taught in each term. The outline term by term is shown in Table 3-2.

**Table 3-1: Summary of Teachers, Students and Schools
Inservice Teachers**

Group A - Inservice

Teacher	School	Number of Students
101	A	26
102	B	28
103	C	63 (2 classes)
104	D	30
105	D	30
106	E	55 (2 classes)
107	F	14
108	G	24
109	H	23
110	I	57 (2 classes)
		N = 350

Materials Only Teachers

Teacher	School	Number of Students
201	A	21
202	B	26
203	G	53 (2 classes)
204	J	28
205	K	28
206	L	30
207	H	23
208	E	58 (2 classes)
209	M	32
210	G	22
		N = 321

Table 3-2: Geometry Objectives**Term 1 - Objectives**

September 15 to November 30

1. Classify and identify polyhedrons.
2. Identify relationships of edges and faces in polyhedrons.
3. Construct regular polyhedrons from nets.
4. Sketches of 3-D objects.
5. Classify angles by size.
6. Properties of intersecting, perpendicular and parallel lines.
7. Classify triangles by sides, angles and symmetry.
8. Classify quadrilaterals by sides, angles, and symmetry.
9. Identify congruent figures and their properties.

Term 2 - Objectives

December 1 - February 28

10. Line symmetry in plane figures.
11. Rotational symmetry in plane figures.
12. Construct geometric figures.
13. Symmetry and congruency in designs.
14. Recognize and draw lines of symmetry.
15. Graph ordered pairs.

Term 3 Objectives

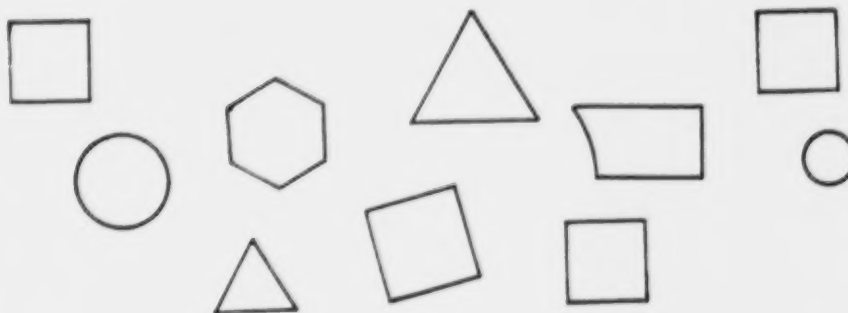
March 1 to May 30

16. Slides, flips and turns.
17. Create tiling patterns.
18. Properties of tiling patterns.
19. Basic properties of translations, reflections and rotations.
20. Draw images under translation, reflection and rotation.
21. Solve tiling puzzles.
22. Enlargements and reductions.

3.4. Instrumentation: Students

Two evaluation booklets containing parallel objective referenced items were designed for the purpose of evaluating student achievement in relation to the geometry objectives. Each form contained 24 geometry items. The items were similar in form to the following:

G9 - Mark an "X" on all the shapes that are congruent.



The booklets also contained 6 measurement items and 10 numerical items to allow for a comparison between geometry and other content in the curriculum. Each objective was identified by a number (G9), the concept was matched to the testing item (□, □, □, all congruent) and each student answer could be scored right or wrong. Items used in the study matched items that had previously been used in five years of matrix sampling program evaluation within The Peel Board of Education. The printed booklets were also edited and validated by three mathematics consultants; two who worked in The Peel Board of Education schools and one who worked for another board of education.

3.5. Teacher Questionnaires

Prior to the final inservice on April 6, 1989 two teacher questionnaires were designed by the author of this study (see Appendix F). The questions were also reviewed by a Consultant in the board Research Unit. Each questionnaire was designed to allow both groups of teacher participants to give their opinions relative to the geometry study.

The questionnaire for the first group contained five items related to inservice which included the following factors: timing and length, activities, structures/properties, experiences and teaching, and other suggestions. Non-inservice teachers did not receive the inservice questions.

Both teacher groups responded to four questions related to the prepared instructional materials. These questions included: appropriateness of materials, structures/properties, materials and teaching, and teacher suggestions. Two questions on the evaluation booklets were included for each group and all respondents were given an open-ended opportunity to comment on the geometry study in general.

Inservice teachers completed the questionnaire at the April 6 inservice session. Questionnaires were mailed to non-inservice teachers by the first week of April to be returned by April 30, 1989.

3.6. Teacher Inservice

As previously outlined the teachers in the study were divided into two groups. One group received inservice and student instructional materials. A second group received only the student instructional materials. Both groups were asked to teach the same geometry objectives as outlined across three terms of the school year 1988-89. The Coordinator of Mathematics who authored this study designed the student instructional materials (see Appendix E) and directed the inservice sessions.

The inservice group participated in three inservice sessions on September 16, 1988; January 15, 1989; and April 6, 1989. The sessions took place at The Peel Board of Education offices and were 2.5 hours in duration beginning at 4 p.m. after the end of the school day.

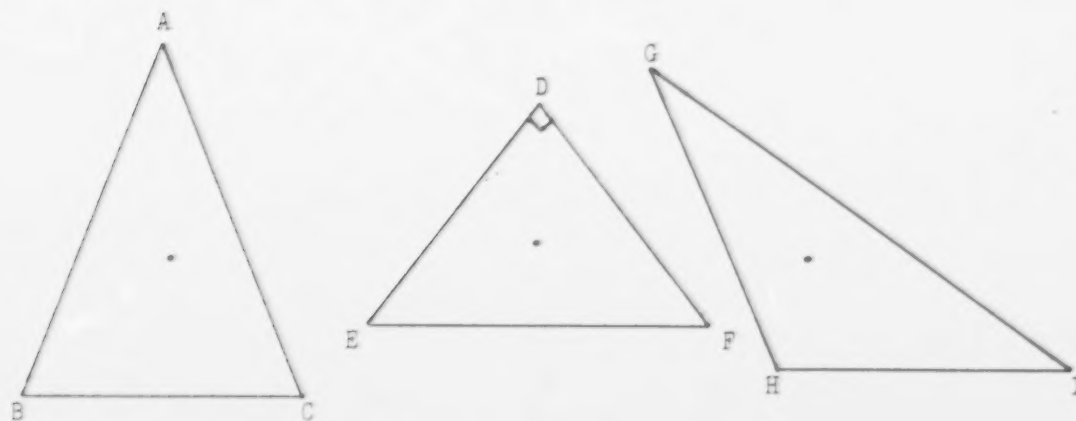
The teachers in the inservice sessions were introduced to the student

instructional materials by actually participating in activities which were designed to deliver geometry objectives to their students. During the inservice the Coordinator of Mathematics analyzed the properties and structures that he had identified as important for the teaching of students.

Two sample activities and the structures are included here as reference examples:

Objective: Classify triangles by sides, angles and symmetry.

Activity: Given three cardboard triangles:



COMPLETE THIS CHART

	$\triangle ABC$	$\triangle DEF$	$\triangle GHI$
ANGLE		RIGHT	
SIDES	EQUAL (3)		
			SCALENE
SYMMETRY			NO LINES
		TURN 360°	

Structures and properties to be emphasized:

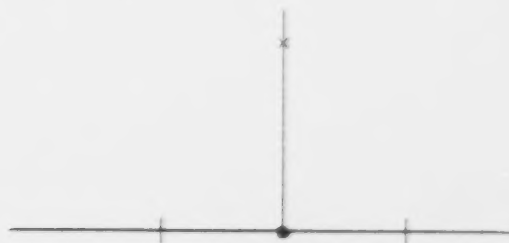
ANGLE	—————→	RIGHT, ACUTE OR OBTUSE?
SIDES	—————→	EQUAL(2), EQUAL(3), OR NOT EQUAL? ISOSCELES, EQUILATERAL, OR SCALENE?
SYMMETRY	—————→	ONE LINE, THREE LINE, OR NO LINES? TURN 180° , TURN 60° , OR TURN 360° ?

Objective: Construct a perpendicular to a line.

Activity:

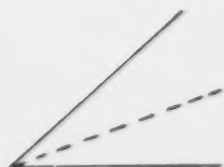


From a point on a given line use the compasses to mark two points equal distances from the chosen point. Using the two marked points as centres describe two arcs that cross above the given line. From a chosen point on the line draw a line passing through the point where the two arcs intersect.



Structures and properties to be emphasized:

Arcs - two equidistant from original point.



Perpendicular - all points on the perpendicular are equidistant from the two arcs. Extend this concept to the bisecting of an angle.

Points equidistant from each arm of the angle.

Activities similar to the above were used in each inservice session and subsequently delivered to all teachers in both groups as a package of teaching materials for students.

3.7. Data Collection

Students in the study were pretested before September 15, 1988 using booklet #1 of the evaluation forms (see Appendix D for a complete evaluation form and teacher instructions). A post-test using booklet #2 was administered to each student before May 30, 1989.

The completed student tests were marked by a team of markers directed by the author. Using a standard marking scheme prepared in advance the author marked two randomly selected papers from the total sample. Each of four markers, all university students, were then assigned the task of marking the same two papers. The results of the marking were similar. However there were some minor differences which were eliminated by discussion. Pre-tests and post-tests were marked in a similar manner.

Student results were reported as right or wrong for each objective. This generated a p-value (percentage correct) for each item tested. Results were also analyzed under the three sections of geometry, measurement, and number.

Teachers were identified by a code number (101, 201) and this generated data class by class as well as school by school. Students were identified by school, class and gender to allow for comparative analysis. The Peel Board Research Department also provided student scores on the Canadian Achievement Test. These tests were administered to all grade 7 students in March, 1989. The results of these tests provided a standardized baseline measure for all students in the study.

3.8. Data Analysis

The data were entered onto a computer and SPSSPC was utilized to perform various analyses. Students' final answers on both the pre-test and post-test were tabulated as right or wrong. These tabulations were utilized to obtain a p-value for each objective being evaluated.

For the separate categories of geometry, measurement, and numerical methods the resulting p-values were analyzed to determine the number of objectives > 80 and the number of objectives < 50 . Objectives with extremely high or low p-values were also identified.

Using a paired t-test pre and post-test comparisons were done in each of the three content categories for both the inservice groups student results and the non-inservice students results. T-tests were also used to compare results from the inservice group and the results from the non-inservice group on the pre-test. The post-test results were compared in the same way.

Gender differences were also analyzed by using a t-test. Pre-test results and post-test results for inservice and non-inservice were examined.

Information gathering questionnaires were distributed to both groups of teachers. Inservice teachers received the questionnaires at the final inservice sessions and questionnaires were mailed to non-inservice teachers. The questionnaires were analyzed for teacher responses and comments. Responses were tabulated and all comments were included in the study.

Chapter 4

The Results

The purpose of this study was to examine relationships between teacher inservice, geometry instructional materials and student achievement in geometry at the grade 7 level. As outlined in Chapter 3, the inservice group consisted of ten teachers (101-110) who taught 350 students. The non-inservice group was made up of ten teachers (201-210) and 321 students. Table 4-1 lists inservice and non-inservice teachers by number and also illustrates the number of students who completed the September, 1988 pre-test and the May, 1989 post-test. The number of students in both groups respectively were 317 and 266 for a total sample population of 583. The pre-test was administered to 671 students but there were 88 fewer students participating in the final testing. Some students were transferred to other classes and some students left the schools.

Table 4-2 illustrates the student sample by gender and group. The number of students categorized as male and female was confirmed by the teachers in May, 1989 and adjustments were made to obtain final totals.

In Table 4-3, the mean national percentile score for each class from Canadian Achievement Test results are listed. Each student in the study completed the mathematics battery of the Canadian Achievement Test in April, 1989. The administration of the CAT was supervised by the board research department and scoring was contracted to the publisher of the test. The scores in the table were obtained from printouts provided to the board as illustrated in Appendix G.

Table 4-1: Number of Students per Teacher

Inservice		Non-Inservice	
Teacher ID	Students	Teacher ID	Students
101	23	201	16
102	26	202	24
103	60	203	47
104	24	204	24
105	26	205	22
106	50	206	19
107	9	207	18
108	23	208	50
109	23	209	25
110	53	210	21
n = 10	n = 317	n = 10	n = 266

Table 4-2: Student Population by Gender

	Inservice Group	Non-Inservice Group
Female	158	142
Male	159	124
	n=317	n=266

From Canadian Achievement Test results the inservice group percentiles were compared to non-inservice group percentiles. These results are summarized in Table 4-3. An examination of this Table reveals that the two groups are fairly similar in their performance on this standardized test.

4.1. Pre and Post-Test p-Values

Each evaluation item was matched to a specific objective. This allowed the calculation of a percentage correct value (p-value) from 0 to 100 to measure student achievement on each separate teaching objective. There were 24 objective based items for geometry, 6 for measurement and 10 for numerical methods. An evaluation item was scored as right(R) or wrong(W) by an analysis of student responses. Table 4-4 summarizes the mean percentages of right and wrong responses for the student sample from the two teacher groups in both the pre-test and the post-test.

4.2. Geometry

For students from both teacher groups the percentage value (p-value) increased for geometry objectives from the pre-test to the post-test with the exception of 2 item scores from the non-inservice group. Scores on G11 - identifying rotational symmetry and G12(13) - constructions decreased by 9.8 and 21.4% respectively within the non-inservice group.

Objectives with a p-value less than 50 for the inservice group in May, 1989 included: G1A - identifying 3-D models, G8(9)A - identifying quadrilaterals, G8(9)B - symmetry in quadrilaterals, G20(23)A and G20(23)B - slide images. Along with the 5 objectives listed above the non-inservice group also had G11 - rotational symmetry, G12(13) - constructions, and G20(25) - turn images below the 50% value.

The inservice group had eleven objectives from the post-test with p-values

Table 4-3: Canadian Achievement Tests

Teacher	Inservice	Teacher	Non-Inservice
	National Percentile		National Percentile
101	66	201	77
102	56	202	60
103	83	203	63
104	38	204	39
105	40	205	35
106	51	206	36
107	42	207	56
108	52	208	41
109	74	209	42
110	41	210	65

**Table 4-4: P-values Pretest and Posttest by Objective:
Inservice and Non-inservice**

		In- service R	1988 W	In- service R	1989 W	Non-In- service R	1988 W	Non-In- service R	1989 W
Geometry									
G1A	Identify 3-D Models	25.9	74.1	46.1	53.9	23.7	76.3	27.1	72.9
G1B	Faces and Edges	32.8	67.2	62.1	37.9	25.6	74.4	57.1	42.9
G2A	Congruent Faces	36.3	63.7	82.0	18.0	24.1	75.9	65.8	34.2
G2B	Parallel Edges	25.6	74.0	70.7	29.3	19.2	80.8	64.7	35.3
G5(6)	Classify Angles	73.2	26.8	88.6	11.4	74.8	25.2	83.5	16.5
G6(7)	Lines	36.3	63.7	95.0	5.0	30.5	69.5	92.5	7.5
G78A	Triangle-Angles	66.6	33.4	79.8	20.2	69.5	30.5	78.9	21.1
G78B	Triangle-Sides	42.6	57.4	80.1	19.9	42.1	57.9	63.2	36.8
G89A	Identify Quadrilateral	16.4	83.6	47.3	52.7	16.5	83.5	38.3	61.7
G89B	Quadrilateral Symmetry	29.3	70.7	45.1	54.9	26.3	73.7	41.0	59.0
G910	Congruent Circles	15.1	84.9	86.1	13.9	10.2	89.8	64.3	35.7
G10	Triangle Symmetry	81.7	18.3	93.1	6.9	80.8	19.2	89.5	10.5
G11	Symmetry Rotational	48.9	51.1	54.6	45.4	44.0	56.0	34.2	65.8
G1213	Constructions	65.6	34.4	71.0	29.0	62.4	37.6	41.0	59.0
G1415	Symmetry Line	67.2	32.8	98.4	1.6	65.8	34.2	88.7	11.3
G1516	Grid Coordinates	29.7	70.3	71.9	28.1	27.8	72.2	59.4	40.6
G1718	Flip,Slide,Turn	69.7	30.3	91.8	8.2	57.1	42.9	71.1	28.9
G1820	Symmetry Line	57.4	42.6	88.6	11.4	50.8	49.2	70.7	29.3
G1922	Reflection Image	83.9	16.1	92.7	7.3	88.3	11.7	89.8	10.2
G2023A	Slide Image	19.9	80.1	44.8	55.2	13.5	86.5	35.0	65.0
G2023B	Slide Image	12.3	87.7	41.0	59.0	10.2	89.8	28.2	71.8
G2024	Reflection Image	78.2	21.8	92.7	7.3	73.7	26.3	85.3	14.7
G2025	Turn Image	16.4	83.6	54.6	45.4	8.3	91.7	42.1	57.9
G2126	Line Symmetry	28.1	71.9	68.8	31.2	22.9	77.1	62.4	37.4

greater than 80 while non-inservice only had 6 in the same category. G2A - congruent faces, G7(8)B - classifying triangles by sides, G9(10) - congruent circles, G17(18) - slide, flip, turn and G18(20) - line symmetry were all higher than 80 for the inservice group but not for non-inservice.

The highest p-value for the inservice group was for G14(15) - line symmetry at 98.4 and the lowest p-value was 41.0 for G20(23)B - slide images. For non-inservice the highest p-value was for G6(7) - lines at 92.5 and the lowest p-value was for G1A - identifying 3-D models at 27.1.

For all geometry objectives the p-values for inservice were greater than the p-values for non-inservice on the May, 1989 post-test.

As anticipated both groups gained in achievement from the pre-test to the post-test. A gain was predictable because each student had a year of instruction between the tests. Most of the objectives with lower p-values including rotational symmetry and constructions are objectives newly introduced at this grade level. When you consider the relative opportunity to learn for new grade level objectives lower p-values would not be unusual.

4.3. Measurement

The first 5 out of 6 p-values for measurement objectives increased from the pre-test to the post-test. For M11D - relationships between the litre and millilitre the p-value was lower in May, 1989 for both groups. In the inservice group there was a decrease of 7.3 and in non-inservice a decrease of 7.5.

For the inservice group objective M11A - cubic decimetre and objective M11D - litre/millilitre had p-values less than 50. Objectives with p-values less than 50 for non-inservice included: M7 - area, M11A - cubic decimetre, and M11D - litre/millilitre.

Both inservice and non-inservice had only one objective with a p-value greater than 80. M11B - metre centimetre was above 80 in both samples.

The highest p-value for the inservice group was for M11B - metre/centimetre at 82.0 and the lowest p-value was for M11A - cubic decimetre at 23.0. For non-inservice the highest p-value was M11B - metre/centimetre at 80.8 and the lowest p-value was for M11A - cubic decimetre at 11.3. The p-values for M11A 23.0 and 11.3 were the only p-values less than 25 in the whole sample.

For all measurement objectives the p-values for inservice were greater than the p-values for non-inservice on the post-test. However, p-values for measurement objectives were generally lower than for geometry objectives. Scores for M11D litre/millilitre and M11A cubic decimetre would seem to indicate that students had little experience with these concepts.

4.4. Numerical Methods

All p-values for numerical methods increased from the pre-test to the post-test. Objectives N3C - multiplication, N5B - addition, decimals and N5D - subtraction, decimals had slightly higher p-values in the non-inservice responses.

The inservice group had two objectives below 50. N3C - multiplication and N5C - multiplication, decimals scored at 47.6 and 48.9 respectively. Objectives with p-values less than 50 for non-inservice included: N1B - place value, decimals at 40.6, N3C - multiplication at 49.6, N5A - division, decimals at 47.0 and N5C - multiplication, decimals at 42.9.

The single objective with p-value greater than 80 for both groups was N3A - addition. Inservice scored 91.8 and non-inservice 89.8. These p-values were also the highest values for each group in the numerical methods results. The lowest value for inservice was N3C - multiplication at 47.6 and for non-inservice the lowest-p-value was N1B - place value decimals at 40.6.

In summary, it appears that students are experiencing success with

**Table 4-4: P-values Pretest and Posttest by Objective:
Inservice and Non-inservice**

		In- service R	1988 W	In- service R	1989 W	Non-In- service R	1988 W	Non-In- service R	1989 W
Geometry									
G1A	Identify 3-D Models	25.9	74.1	46.1	53.9	23.7	76.3	27.1	72.9
G1B	Faces and Edges	32.8	67.2	62.1	37.9	25.6	74.4	57.1	42.9
G2A	Congruent Faces	36.3	63.7	82.0	18.0	24.1	75.9	65.8	34.2
G2B	Parallel Edges	25.6	74.0	70.7	29.3	19.2	80.8	64.7	35.3
G5(6)	Classify Angles	73.2	26.8	88.6	11.4	74.8	25.2	83.5	16.5
G6(7)	Lines	36.3	63.7	95.0	5.0	30.5	69.5	92.5	7.5
G78A	Triangle-Angles	66.6	33.4	79.8	20.2	69.5	30.5	78.9	21.1
G78B	Triangle-Sides	42.6	57.4	80.1	19.9	42.1	57.9	63.2	36.8
G89A	Identify Quadrilateral	16.4	83.6	47.3	52.7	16.5	83.5	38.3	61.7
G89B	Quadrilateral Symmetry	29.3	70.7	45.1	54.9	26.3	73.7	41.0	59.0
G910	Congruent Circles	15.1	84.9	86.1	13.9	10.2	89.8	64.3	35.7
G10	Triangle Symmetry	81.7	18.3	93.1	6.9	80.8	19.2	89.5	10.5
G11	Symmetry Rotational	48.9	51.1	54.6	45.4	44.0	56.0	34.2	65.8
G1213	Constructions	65.6	34.4	71.0	29.0	62.4	37.6	41.0	59.0
G1415	Symmetry Line	67.2	32.8	98.4	1.6	65.8	34.2	88.7	11.3
G1516	Grid Coordinates	29.7	70.3	71.9	28.1	27.8	72.2	59.4	40.6
G1718	Flip,Slide,Turn	69.7	30.3	91.8	8.2	57.1	42.9	71.1	28.9
G1820	Symmetry Line	57.4	42.6	88.6	11.4	50.8	49.2	70.7	29.3
G1922	Reflection Image	83.9	16.1	92.7	7.3	88.3	11.7	89.8	10.2
G2023A	Slide Image	19.9	80.1	44.8	55.2	13.5	86.5	35.0	65.0
G2023B	Slide Image	12.3	87.7	41.0	59.0	10.2	89.8	28.2	71.8
G2024	Reflection Image	78.2	21.8	92.7	7.3	73.7	26.3	85.3	14.7
G2025	Turn Image	16.4	83.6	54.6	45.4	8.3	91.7	42.1	57.9
G2126	Line Symmetry	28.1	71.9	68.8	31.2	22.9	77.1	62.4	37.4

**Table 4-5: P-values Pretest and Posttest by Objective:
Intervice and Non-inservice**

		In- service R	1988 W	In- service R	1989 W	Non-In- service R	1988 W	Non-In- service R	1989 W
		Measurement							
M1	Perimeter	51.7	48.3	74.1	25.9	51.1	48.9	65.0	35.0
M7	Area	10.4	89.6	56.5	43.5	7.5	92.5	46.2	53.8
M11A	Cubic Decimetre	4.4	95.6	23.0	77.0	3.0	97.0	11.3	88.7
M11B	Metre-Centimetre	77.3	22.7	82.0	18.0	69.5	30.5	80.8	19.2
M11C	Kilometre-Metre	41.3	58.7	57.4	42.6	37.2	62.8	52.6	47.4
M11D	Litre Millilitre	56.8	43.2	49.5	50.5	51.5	48.5	44.0	56.0

Table 4-6: P-values Pretest and Posttest by Objective:
Intervice and Non-inservice

		In- service R	1988 W	In- service R	1989 W	Non-In- service R	1988 W	Non-In- service R	1989 W
Numerical Methods									
N1A	Place Value	73.5	26.5	77.3	22.7	68.4	31.6	79.2	24.8
N1B	Place Value Decimal	45.7	54.3	69.4	30.6	39.8	60.2	40.6	59.4
N3A	Addition	79.8	20.2	91.8	8.2	78.9	21.1	89.8	10.2
N3B	Subtraction	58.0	42.0	74.4	25.6	51.5	48.5	66.7	33.1
N3C	Multiplication	33.8	66.2	47.6	52.4	28.2	71.8	49.6	50.4
N3D	Division	50.5	49.5	58.0	42.0	38.3	61.7	57.1	42.9
N5A	Division Decimals	25.6	74.4	53.0	47.0	14.7	85.3	47.0	53.0
N5B	Addition Decimals	44.5	55.5	62.8	37.2	30.8	69.2	63.2	36.8
N5C	Multiply Decimals	33.8	66.2	48.9	51.1	22.6	77.4	42.9	57.1
N5D	Subtract Decimals	53.6	46.4	53.6	46.4	35.3	64.7	54.5	45.5

**Table 4-7: Paired t-test: Pre-test and Post-test
Geometry - Inservice**

	n	M	S	t	p
September 1988	317	10.59	4.068	-29.15	<.001
May 1989	317	17.27	3.950		

**Table 4-8: Paired t-test: Pre-test and Post-test
Geometry - Non-Inservice**

	n	M	S	t	p
September 1988	266	9.70	3.518	-21.65	<.001
May 1989	266	14.71	4.221		

The means, standard deviations and results of a paired t-test for numerical methods are illustrated in Tables 4-11 and 4-12. The results reveal a significant difference between the pre-test and the post-test ($p < .001$). Unlike geometry and measurement the non-inservice materials only group scores increased. These are illustrated in Figure 4-1.

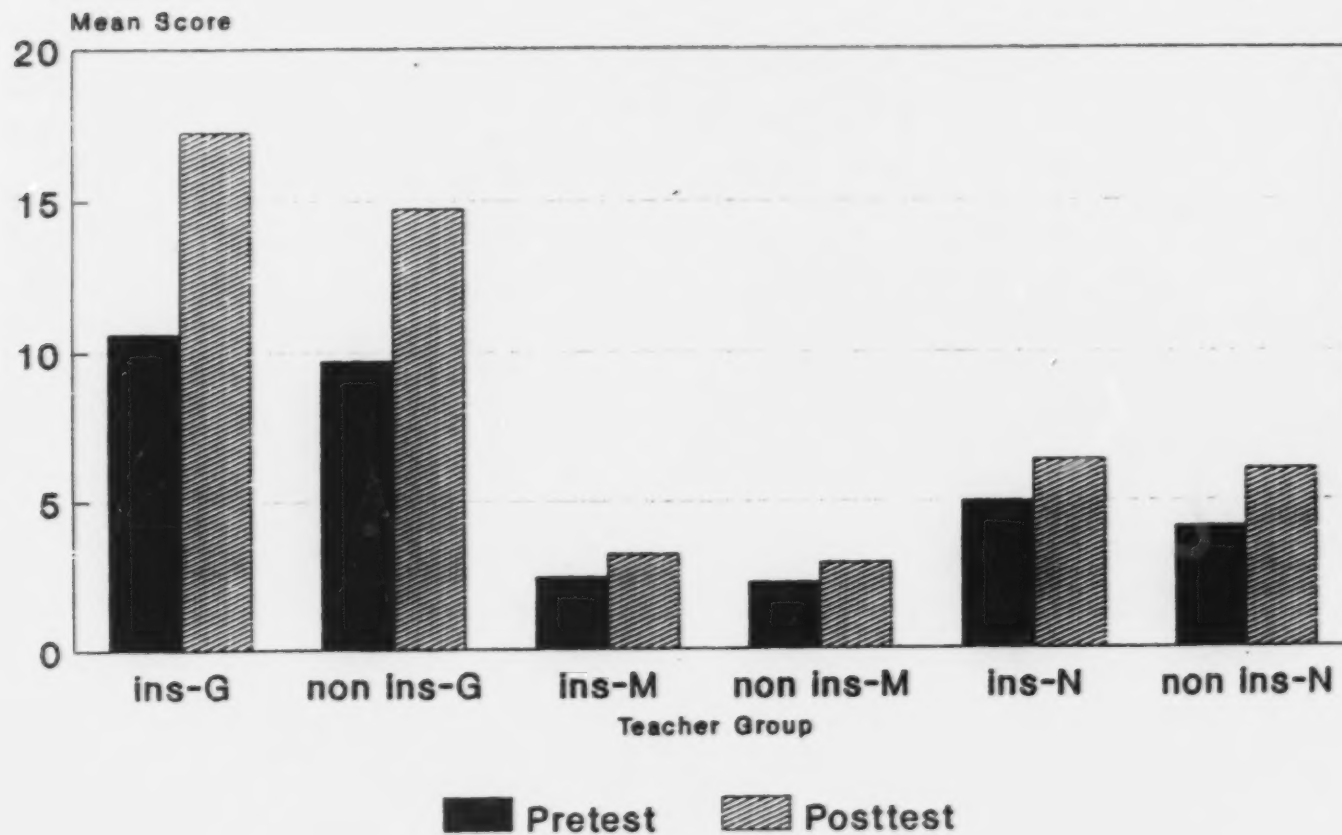
**Table 4-9: Paired t-test: Pre-test and Post-test
Measurement - Inservice**

	n	M	S	t	p
September 1988	317	2.42	1.409	-8.87	<.001
May 1989	317	3.20	1.668		

**Table 4-10: Paired t-test: Pre-test and Post-test
Measurement - Non-inservice**

	n	M	S	t	p
September 1988	266	2.21	1.370	-7.90	<.001
May 1989	266	2.88	1.457		

Results showing the means, standard deviations and t-values for the inservice group and non-inservice geometry results on the pre-test and the post-test are shown in Tables 4-13 and 4-14. When the pre-test results were compared the t-value was 2.89 and there was a significant difference between the two groups ($p = .004$). Post-test comparisons yielded a t-value of 7.99 which indicates a significant difference ($p < .001$).



ins=in-service non ins=non in-service

Figure 4-1: Comparisons of Means - Pre + Post-tests
Geometry, Measurement, Numerical Methods

**Table 4-11: Paired t-test: Pre-test and Post-test
Numerical Methods - Inservice**

	n	M	S	t	p
September 1988	317	4.99	2.540	-8.92	<.001
May 1989	317	6.37	2.497		

**Table 4-12: Paired t-test: Pre-test and Post-test
Numerical Methods - Non-inservice**

	n	M	S	t	p
September 1988	266	4.09	2.634	-12.26	<.001
May 1989	266	6.05	2.413		

Measurement test results for pre-test and post-test were also compared. Inservice and non-inservice means, standard deviations and t-values are illustrated in Tables 4-15 and 4-16. The pre-test comparison resulted in a t-value of 1.91 which indicates no significant difference between the means of the two groups ($p > .05$). The post-test t-value was 3.18 and this indicates a significant mean difference for inservice ($p < .01$).

The numerical methods results for the pre-test and the post-test are illustrated in Tables 4-17 and 4-18. There is a significant mean difference between the 2 groups on the numerical methods results ($p < .001$). Both groups

Table 4-13: Pre-test 1988 Geometry between Inservice and Non-inservice

	n	M	S	t	p
Inservice Group	317	10.59	4.068	2.89	.004
Non-Inservice Group	266	9.68	3.523		

Table 4-14: Post-test 1989 Geometry between Inservice and Non-inservice

	n	M	S	t	p
Inservice Group	317	17.47	3.999	7.99	<.001
Non-Inservice Group	266	14.74	4.212		

Table 4-15: Pre-test 1988 Measurement between Inservice and Non-inservice

	n	M	S	t	p
Inservice Group	317	2.42	1.409	1.91	.057
Non-Inservice Group	266	2.20	1.374		

Table 4-18: Post-test 1989 Measurement between Inservice and Non-inservice

	n	M	S	t	p
Inservice Group (100)	317	3.43	1.717	3.18	.002
Non-Inservice Group (200)	266	3.00	1.512		

Table 4-17: Pre-test 1988 Numerical Methods between Inservice and Non-inservice

	n	M	S	t	p
Inservice Group (100)	317	4.99	2.540	4.17	.000
Non-Inservice Group (200)	266	4.09	2.629		

Table 4-18: Post-test 1989 Numerical Methods between 100 and 200 Classes

	n	M	S	t	p
Inservice Group (100)	317	6.37	2.497	1.54	.125
Non-Inservice Group (200)	266	6.06	2.409		

Table 4-19: Means, Standard Deviations and t-values Geometry, Measurement and Numerical Methods Inservice Group by Student Gender - Pre-test

Pre-test						
Test	Gender	n	M	S	t	p
Geometry	Male	156	10.40	4.294	-.83	.204
	Female	161	10.78	3.841		
Measurement	Male	156	2.53	1.425	1.40	.085
	Female	161	2.31	1.389		
Numerical Methods	Male	156	4.75	2.634	-1.64	.051
	Female	161	5.22	2.431		
Post-test						
Test	Gender	n	M	s	t	p
Geometry	Male	159	17.43	4.255	-.16	.436
	Female	158	17.51	3.736		
Measurement	Male	159	3.38	1.817	-.44	.330
	Female	158	3.46	1.615		
Numerical Methods	Male	159	6.19	2.649	-1.29	.098
	Female	158	6.55	2.328		

Table 4-20: Means, Standard Deviations, and t-values Geometry, Measurement, and Numerical Methods Non-Inservice Student Gender

Pre-test						
Test	Gender	n	M	S	t	p
Geometry	Male	129	9.82	3.539	.63	.263
	Female	137	9.55	3.517		
Measurement	Male	129	2.22	1.425	.20	.419
	Female	137	2.18	1.330		
Numerical Methods	Male	129	3.84	2.561	-1.53	.063
	Female	137	4.33	2.679		
Post-test						
Test	Gender	n	M	s	t	p
Geometry	Male	124	14.60	4.256	-.48	.317
	Female	142	14.85	4.185		
Measurement	Male	124	2.99	1.620	-.08	.46
	Female	142	3.01	1.417		
Numerical Methods	Male	124	5.82	2.502	-1.47	.071
	Female	142	6.26	2.315		

showed growth in mean score on the post-test, however the differences were not significant ($p > .05$).

Inservice and non-inservice results were analyzed for gender differences. The means, standard deviations and t-values for the pre-test and the post-test are shown in Tables 4-19 and 4-20. There were no significant gender differences between the pre-test and the post-test ($p > .05$).

4.6. Teacher Questionnaires

This study had two separate groups of grade 7 teachers. One group received inservice and a second group was excluded from the inservice. Both groups received 3 packages of instructional materials, one for each teaching term of the school-year. Questionnaires were designed to gather data on teacher attitude and opinion about inservice and instructional materials in geometry.

Nine out of ten teachers in the inservice group returned completed questionnaires while only five out of the ten non-inservice teachers returned completed forms. Questionnaires for the non-inservice teachers were delivered by mail with no follow-up personal contact.

Inservice teachers responded very positively to the five questions about the inservice treatment. Results are as follows:

a. *Was the timing and length of the inservice satisfactory?*

All respondents said yes and comments included: "would not have wanted less"; "the first session was a little long"; "hard to find time to attend" and "presentations were efficient and helpful".

b. *Were the inservice activities appropriate?*

The inservice teachers all thought that the activities were very useful or fairly useful and stated: "making notes was helpful"; "I based my teaching around the inservice activities"; "helped to see hands-on"; "I have never enjoyed learning geometry until this inservice".

- c. *Did the inservice sessions clarify the structures and properties of the geometry?*

Most of the participants indicated that the sessions were very useful in clarifying the structures and properties and said: "helped me to know I was on the right track"; "more detail would have helped" and "links between topics were exposed".

- d. *Please reflect on the relationship between the inservice experience and your teaching*

Participants were positive and enthusiastic and commented: "gave life to the curriculum"; "I used the materials as experiments and students were impressed"; "reinforced the hands-on approach"; "helped me focus on timing and sequence of geometry objectives"; "made me feel more comfortable" and "useful examples".

- e. *Are there any suggestions you would like to make about the geometry inservice?*

Suggestions made by the teachers included: "requires more inservice"; "we shouldn't assume that teachers know the terms"; "all teachers should get inservice"; "printed words often need translation"; "do the inservice during the day not after school" and "continue as is".

Both the inservice and the non-service teachers responded to seven identical questions referring to the instructional materials represented in Appendix F. The results are outlined question by question.

- f. *Were the student instructional materials appropriate for the teaching objectives?*

All of the teachers responded positively to this question. One inservice teacher commented: "used all with success" and one non-inservice teacher said: "we need instructions to match objectives".

- g. *Did the instructional materials clarify the structures and properties of geometry for the students?*

On the three point scale very useful, fairly useful, or of little use, ten teachers indicated that the instructional materials were fairly useful. One of the non-inservice teachers felt the materials were of little use.

- h. *Please reflect on the relationship between the student instructional materials and your teaching of geometry*

Most of the teacher comments were positively directed. The inservice teachers stated: "students enjoyed manipulations best"; "these materials formed a base for my teaching"; "had to beg and borrow concrete materials"; "materials got me started"; "enabled me to diversify my math classes"; "acted as a stimulus for ideas which the students could visualize" and "helps with my time management".

Three of the non-inservice teachers commented: "materials did not fit my timing"; "similar to my own ideas" and "fit easily into the structure of my lessons".

- i. *Are there any suggestions you would like to make about student instructional materials?*

Suggestions made by the inservice teachers included: "great, keep it up"; "we need a larger pool of materials"; "include materials in the curriculum guideline"; "more access for teachers" and "more! put them in the school library". The non-inservice teachers were not as supportive when they said: "relate to numbered objectives" and "further clarification is necessary".

- j. *Were the evaluation forms appropriate for the geometry objectives?*

Only ten out of fourteen respondents completed this question and all of these responses were positive.

- k. *Are there any questions you would like to make about the evaluation*

materials?

Only three teachers commented and they were all inservice teachers. The comments were: "good sample packets"; "very professional" and "make them readily available".

Two questions solicited information from only the non-inservice teachers. One question addressed delivery and implementation and the other question inquired about a need for inservice.

l. Were the instructional materials as delivered easily implemented?

One teacher indicated that the materials were very useful, two checked fairly useful and two checked of little use.

m. In your opinion would inservice related to geometry assist you?

Only four teachers responded and two said yes and two said no. One teacher indicated "we need lesson packages" and another said "depends on the kind of inservice".

All of the teachers were given an opportunity to make general comments about the geometry study. Inservice teachers comments included: "glad to be involved"; "I enjoyed participating"; "thank-you for allowing me this valuable experience"; "have enjoyed it" and "hands-on was very valuable". The non-inservice teachers said: "lack of direction was frustrating"; "packages of materials did not meet my needs" and "materials as sent do not make a significant impact".

The data obtained from the teacher questionnaires was very supportive of the need for inservice in geometry although two non-inservice teachers did not see the necessity for inservice. It appears as if successful inservice produces support for more inservice.

Inservice teachers were far more supportive of instructional materials than the non-inservice teachers. Indications are that materials delivered without inservice support will be used less effectively. Personal contact particularly with

the designer of the instructional materials provided an opportunity for both support and clarification.

Chapter 5

Conclusions, Discussion and Recommendations

This chapter concludes the study of the effect of conducting a geometry inservice program and the use of special instructional materials on the mathematics achievement of grade 7 students. Presented here is a summary of the study, conclusions and discussion of the results followed by specific recommendations.

5.1. Summary

The primary purpose of this study was to determine whether inservice for teachers based on the structures and properties of geometry concepts would change the achievement levels of students relative to grade 7 geometry objectives. By teaching teachers about structures and by having teachers teach students using the same structures achievement in geometry should be positively changed.

Two groups of grade 7 teachers volunteered to participate in the study. All of the teachers were employees of a large school board in the Province of Ontario, Canada. The inservice group ($n = 10$) received a two-hour inservice session at the beginning of 3 school terms. The inservice program clarified the structures and properties inherent in instructional materials designed for use with this study. All instructional materials related to the implementing of geometry teaching objectives outlined at the grade 7 level in the Board's Mathematics Curriculum.

The teachers in the non-inservice group ($n = 10$) received the same packages of instructional materials as the inservice group but were excluded from any inservice in geometry. Teachers in both groups gave students the opportunity to learn the same set of objectives.

All students completed an objective based pre-test in September, 1988. In May 1989, a parallel post-test was administered to the same students. The student achievement data for objectives in geometry, measurement and numerical methods were analyzed as percentage scores (p-values) for each individual objective.

Pre-test and post-test results were compared for the three categories of geometry, measurement and numerical methods. The pre and post-test results were also compared for gender differences. The results of the study show significant mean differences in student achievement but no significant gender differences.

In addition to the objective based tests all students also completed the mathematics sections of the Canadian Achievement Test for their grade level. This allowed for a comparison of the two groups based on a norm-referenced standardized test.

Questionnaires soliciting attitudes and opinions about inservice and instructional materials were administered to both teacher groups. Teacher comments were analyzed question by question and general suggestions about the study were recorded.

5.2. Conclusions

Subject to the limitations of the study, the data analysis appears to support these conclusions:

1. The geometry objectives as described in this study can be implemented successfully at the grade 7 level with the exception of rotational symmetry, constructions and formal transformations.
2. Achievement levels for students whose teachers received inservice in geometry were significantly higher but it is not clear whether the inservice on structures and properties created the difference.

3. There are no significant gender differences in achievement at grade 7 when an equal opportunity to learn is present.
4. Hands-on instructional materials are supported by teachers but require inservice for clarification.

5.3. Discussion

This study illustrated the identification of geometry teaching objectives for use with grade seven students in the Province of Ontario. The differences in student achievement on the pre-test and the post-test were significant and indicate that the defined teaching objectives can be implemented by teachers at the grade seven level. Teachers who participated in the study identified a need for geometry inservice. The student achievement (p-values) for geometry objectives for the inservice teachers was greater than achievement on the same objectives for the non-inservice teachers. However, both groups did exhibit increased student achievement.

Teacher comments from questionnaires indicated that inservice combined with teaching materials was more motivating and received with more enthusiasm by the teachers than the materials alone. Several teachers reported the materials alone to be of little use in creating differences in teaching behaviour.

Based on scores from a standardized test it was apparent that all students in the sample exhibited a normal range of mathematical achievement. However, when the pre-test results on geometry, measurement and numerical methods were analyzed the achievement of students of teachers in the inservice group were significantly higher on geometry and numerical methods.

When the results of the pre-test and post-test were analyzed the inservice group improved more in geometry than non-inservice but both groups improved significantly. Both groups also improved significantly in measurement and numerical methods. Inservice improved more in measurement and non-inservice

more in numerical methods. Whether the differences in student achievement can be attributed to the fact that these teachers were in a study is not clear. What is clear given the results in geometry, measurement and numerical methods is that given an opportunity to learn this content, grade 7 students can learn the concepts. Objective M11A, the relationships between the cubic decimetre and the litre, seems to be an example of an objective that teachers are not teaching and therefore students do not get the opportunity to learn. In this study only geometry was included in the inservice and this meant that both inservice and non-inservice teachers were not externally influenced to change teaching behaviours related to measurement or numerical methods objectives.

An informal comparison of the geometry p-values obtained in the study with the p-values from the 1988 Board-wide matrix sampling evaluation proved interesting. Most p-values for students in the study were higher than p-values obtained by grade 7 students in May, 1988. Five objectives; G6(7) - lines, G7(8A) - classify triangles by angle, G7(8b) - classify triangles by sides, G10 - triangle/symmetry and G20(25) - turn angles, showed major increases in achievement. These five objectives were highlighted in the instructional materials.

The student instructional materials were designed specifically to highlight the structures and properties of the geometrical concepts. This was appropriate and is supported in the literature by many mathematics educators. Although the structures were also highlighted during the inservice presentations it is not clear from the results of this study whether this emphasis on structure was significant in improving student achievement in geometry. Students whose teachers did not receive inservice had lower achievement on rotational symmetry, constructions, and transformations. These were highlighted in the inservice presentations but whether structure or better use of the instructional materials made the differences is not confirmed by the results. Inservice teachers did indicate that the sessions helped to clarify structures and properties.

Teacher comments about 3-D materials and student achievement on G1D identifying 3-D models illustrated difficulty in obtaining and using hands-on materials. The results indicate a need for implementation of appropriate hands-on materials and an active learning/evaluation model. Inservice teachers expressed enthusiastic support for the hands-on approach used with them and indicated a transfer of this approach to the practice in the classroom. An emphasis on concrete representation in elementary geometry as outlined by Mayberry (1983) would be supported by this study. The study results also support Wang's (1972) conclusion that both students and teachers should acquire geometric content through an informal, inductive, manipulative approach. Inservice teachers indicated that "students enjoyed manipulatives best".

There is a difference in achievement on geometry for the students whose teachers received inservice. This difference in achievement combined with positive comments from the teachers would lead to the conclusion that inservice does lead to benefits in terms of student learning. Inservice teachers supported the content, the timing, and the need for inservice enthusiastically. Non-inservice teachers also saw a need for inservice.

Hanna (1988) concluded that girls and boys assimilate geometry equally well if it is taught. The results of this study with grade 7 students would indicate that Hanna's conclusions are valid. Male and female students in the study, taught by the same teachers using the same instructional materials, achieved equally well. Also, there were no significant gender differences between inservice and non-inservice groups.

When given an opportunity to respond to a questionnaire, nine of the inservice teachers returned the questionnaire and only five of the non-inservice sent a return. However, the first group were handed the questionnaire at an inservice session while the forms were mailed to the second group. This could account for the difference in the rate of return.

Inservice teachers responded positively to five questions about the inservice. The timing and length of sessions was seen as satisfactory. Teachers viewed the activities as useful and helpful in clarifying structures and properties. The participants commented that the inservice experiences assisted them with the teaching of geometry particularly with hands-on materials and sequencing objectives. Teacher comments reinforced the need to continue providing inservice.

Seven questions referring to the instructional materials were answered by both groups of teachers. The respondents all said that the materials were appropriate for the objectives. One non-inservice teacher felt the need for more instructions. Non-inservice teachers also were doubtful that the materials did clarify structures and had some difficulty fitting the materials into their teaching. Further clarification was requested by non-inservice teachers while inservice teachers enthusiastically supported the materials. This would indicate that delivery of materials without inservice for clarification is not the best implementation strategy.

Responses to questions about the pre-test and post-test outlined in Appendix D were very positive. The evaluation items were designed to match the model used in the annual program evaluation sample done for the Board. Teachers seemed familiar with the technique and responded positively.

Non-inservice teachers answered 2 questions not presented to inservice teachers. When asked about the ease of implementing the delivered instructional materials 3 out of 5 teachers responded negatively. This response would again support the need for inservice to clarify the use of instructional materials.

The non-inservice responses to the need for inservice was mixed. Two respondents recognized a need, one said, "the need depended on the kind of inservice" and two did not see a need for inservice. This response coming from non-inservice teachers leads to the conclusion that when inservice is done

properly the participants will support more inservice. Inservice teachers all asked for a continuation of inservice.

The results obtained in this research have implications for the teaching of geometry in the elementary school. The findings suggest that it is important for teachers to have a well defined and sequenced set of objectives. Instructional materials designed for specific objectives can best be implemented when accompanied by inservice training. Hands-on visual materials positively affect the attitudes about geometry for both teachers and students. When given the opportunity to learn on an equal basis there are no differences in achievement between boys and girls.

5.4. Need for Further Research

What is not clear from the findings presented here is whether the structures and properties emphasized in the inservice and materials were a factor in improving student achievement. Teachers expressed appreciation for clarification of structures and properties but more research is required to study the influence of emphasizing structures on student learning. Possibly a pre and post-test based solely on a knowledge of structures and properties would be appropriate.

More research is also required on the use of visual and descriptive instructional materials to teach geometry. The findings support hands-on learning but do not answer the important questions related to which materials are most appropriate for teaching geometry in the elementary school.

From this study, it is evident that elementary teachers need more knowledge about the geometry they are expected to teach and more assistance to teach the geometry. However, more research is required to determine what information teachers need and what they need to enhance the implementation of geometry objectives.

5.5. Recommendations

Based on the evidence presented in the study and the discussion outlined above the following recommendations would appear to be appropriate:

1. The Coordinator of Mathematics for the Board of Education should review and revise the set of teaching objectives for grade 7 geometry particularly in the difficult areas of transformations, rotational symmetry and constructions.
2. The Board should provide inservice and instructional materials for teaching geometry for all elementary teachers.

5.6. Limitations

The study presented here is limited to one large school board, a specific set of geometry objectives and a specific set of grade 7 teachers and students. The results may not be transferable to another jurisdiction, another grade level or to different geometry content. A pure experimental design was not used in this study but some data was analyzed quantitatively.

In addition, this study is described as an instructional study which does not purport to generalize the findings but rather develop an inservice model for the teaching of geometry. The inservice model could be generalized to other school boards.

Teachers included in this study all volunteered and this sample of convenience may not be a representative sample. The enthusiasm of the volunteers may have contributed to growth in student achievement.

A specific set of teaching objectives were defined. All teachers in the study were asked to implement these objectives. However, there were no controls in the study to determine if each individual maintained the defined objectives. The student achievement data appears to indicate that the objectives were implemented.

The pre-test and post-test items matched to common teaching objectives. Items mirrored the Board matrix sampling models and were reviewed by internal and external consultants. However, no item analysis was applied to the student evaluation questions.

The study clearly defines the continuing need, previously identified by Coxford (1978) and Bishop (1983), for more research on the teaching of geometry particularly in the elementary school. Specific research directions are identified in the study as is support for teaching geometry and this it seems can be considered as a modest academic contribution to mathematics education for students.

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Appendix A

Memo to Principals Asking for Teacher Volunteers

PEEL
BOARD OF
EDUCATION

P R O G R A M D E P A R T M E N T

January 20, 1988

TO: Barry Fendley, Principal
Alloa Public School

Included as a part of the 1988-1990 Peel Mathematics Review, we would like to do a study on the relationship between the teaching of geometry and teacher in-service. This study will include grade 7 teachers and their students.

I am soliciting your co-operation to get teacher volunteers from your school. If you wish your school to participate, please distribute the enclosed teacher letters to the grade 7 teachers and submit the names of teacher volunteers to me by the end of January if possible. I realize that school organizations will change for the school-year 1988-89 but wherever possible, volunteers should expect to teach grade 7 mathematics beginning in September, 1988.

At the end of the study, all teachers involved will receive a report on the results and all schools will receive a tested package of geometry teaching materials for use with all teachers by September, 1989.

Thank-you once more for your co-operation.

Alexander L. Norrie
Co-ordinator of Mathematics

Encl.

c: Dr. K. Kirkwood
Chief Research Officer

DLW

Appendix B

Memo to Grade 7 Teachers Describing the Study

January 20, 1988

TO: Grade 7 Mathematics Teachers

FROM: Alexander L. Norrie
Co-ordinator of Mathematics

Across North America, the teaching of geometry in the elementary schools has become a debatable curriculum issue.

You may wish an opportunity to become a part of a research study within the Peel Board, which will contribute data on the curriculum debate about geometry.

We need at least 30 teacher volunteers at the grade 7 level, to study relationships between teacher in-service and student achievement on Peel grade 7 geometry objectives.

If you volunteer, you may be assigned to one of the following groups:

- Group A - will receive 3 complimentary in-service sessions on geometry - one per term
 - will receive instructional materials to teach each term's geometry objectives
 - will be asked to teach each term's core objectives
- Group B - will receive instructional materials to teach each term's geometry objectives
 - will be asked to teach each term's core objectives
- Group C - will be asked to teach each term's core objectives

All teachers in the study will receive a report on the data, as well as a package of tested instructional materials to use in your classroom the following year.

The study will take place in school-year 1988-89 and will include the following events:

- your students will be tested on grade 6 geometry, June 1988
- your students will be tested on grade 7 geometry, September, 1988
- you will do a teacher test and attitude scale early in the school-year, at your convenience
- your students will be tested on grade 7 and 8 geometry, June, 1989.

If you wish to be involved, complete the tear-off slip below and return it to your Principal before the end of January.

Thank-you for your co-operation.

Name _____ School _____

I would appreciate being included in the Peel Grade 7 Geometry Study, with the understanding that I may be required to attend three in-service sessions as well as use specific student instructional materials based on the Peel Core Curriculum objectives.

PRINCIPAL - PLEASE, RETURN TO 'CO-ORDINATOR OF MATHEMATICS'

BY THE END OF JANUARY, 1988

Appendix C
Geometry Objectives Term by Term

May 12, 1988

TO: GEOMETRY STUDY PARTICIPANTS

FROM: Alexander L. Norrie
 Co-ordinator of Mathematics

Enclosed you will find the final listing of participants (hopefully) and an outline of the objectives. These objectives should be included in any long-term planning you will do for next year.

It would be appreciated if between now and the end of June you would take the time to find out which textbook you will be using next year. Send me a little handwritten note with your name and the textbook title. Thanks!

For those of you in group A, I anticipate having the first in-service session between 4 and 6 p.m. at the H.J.A. Brown Education Centre on Thursday, September 15.

The pre-test for students in September will be delivered for administration on Monday, September 12.

Please do not hesitate to call 890-1099, Ext. 2546 if you have any questions.

Encl.

DLW

GRADE 7 - GEOMETRY STUDY TEACHERS - 1988 - 04 - 14

GROUP A

This group will receive in-service each term and teaching materials.

Tom Morrison - Hillside Sr.
Wendy McCausland - Erin Mills Sr.
Sue Checkeris - Dolphin Sr.
Peter Pinfold - Gordon Graydon Sr.
Jackie Wright - Wm. G. Davis Sr.
Bob Boyes - Alloa
Randy Cook - Balmoral Dr. Sr.
Mike Howell - Credit View
Tom Higginbottom - Caledon Central
Ann Shaffer - Tomken Sr.
Heather Deslippe - Earnscliffe Sr.
Linda Lindsay - Greenbriar Sr.

GROUP B

This group will receive only the teaching materials.

Bruce Smith - Hillside Sr.
Bob Sinclair - Erin Mills Sr.
D. Burgess - Green Glade Sr.
Heather Crase - Tomken Sr.
Dale White - Williams Parkway Sr.
Dave Richardson - Wm. G. Davis Sr.
Jagan Dhawan - Darcel Ave. Sr.
Mike Kozak - The Woodlands
Linda Martin - Earnscliffe Sr.
Doug Logan - Balmoral Dr. Sr.
Liz Hunter - Greenbriar Sr.
Peter Klausen - Macville

September - all students will receive a pre-test by the middle of September.

September 15
to November 30

TERM 1OBJECTIVES

1. Classify and identify polyhedrons.
2. Identify relationships of edges and faces in polyhedrons.
3. Construct regular polyhedrons from nets.
4. Sketches of 3-D objects.
5. Classify angles by size.
6. Properties of intersecting, perpendicular and parallel lines.
7. Classify triangles by sides, angles and symmetry.
8. Classify quadrilaterals by sides, angles and symmetry.
9. Identify congruent figures and their properties.
10. Line symmetry in plane figures.
11. Rotational symmetry in plane figures.

December 1
to February 28

TERM 2OBJECTIVES

1. Construct geometric figures.
2. Symmetry and congruency in designs.
3. Recognize and draw lines of symmetry.
4. Graph ordered pairs.

March 1
to May 30

TERM 3OBJECTIVES

1. Slides, flips, and turns.
2. Create tiling patterns.
3. Properties of tiling patterns.
4. Basic properties of translation, reflection and rotation.
5. Draw images under translation, reflection and rotation.
6. Solve tiling puzzles.
7. Enlargements and reductions.

Appendix D
Evaluation Booklet and Teacher Instructions

GEOMETRY STUDY - PEEL BOARD OF EDUCATION 1988-1989

SEPTEMBER - EVALUATION INSTRUCTIONS

All students should take Geometry Test #1, Grade 7 on Monday, September 12th or Tuesday, September 13th if possible. If these two days are not convenient please administer the test no later than Thursday, September 15.

The test does not have an exact time limit. Encourage the students to try every question and use your judgement to determine when most students have completed the paper.

STUDENTS WILL NEED CENTIMETRE RULERS AND COMPASSES.

Questions #1 and #2 require access to 3-D models.

For #1 set out seven models each labelled with a letter as follows:

- A - rectangular prism (4 rectangular sides, 2 squares)
- B - square based pyramid
- C - hexagonal prism (6 rectangular sides, 2 hexagons)
- D - cone
- E - triangular prism (3 rectangular sides, 2 triangles)
- F - tetrahedron
- G - triangular based pyramid

For #2 place one or two square based pyramids on a flat surface.

During the testing period each student should be allowed access to the materials to do question G1 and G2. Resource Teachers can be contacted for assistance.

DO NOT MARK THE STUDENT'S PAPERS.

Check each paper for the inclusion of your name and the student's name. Return all completed papers to:

Joan Thomson
Program Department
Mathematics
H.J.A. Brown Education Centre

Each teacher in the geometry study will receive a report of the evaluation results. This will be done objective by objective with no students or teachers being identified.

Remember also that this is a pre-test on objectives that you will teach during the year. We are not expecting and should not expect students to have mastery of the content at this time.

Thank you for your continued cooperation. I can be reached at 858-4325 if difficulties arise.

Alexander L. Norrie

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

Teacher _____

Student _____

GRADE 7

TEST 1

- G 1 (a) Go to the centre with 3-D models.
Write down the letter for each prism. _____
- (b) Name the prism with 5 faces. _____

R W

R W

- G 2 Use a model of a square based pyramid.

(a) Trace two congruent faces.

R W

(b) Trace two perpendicular edges.

R W

DO THE TRACING ON THE BACK OF THIS SHEET.

- G 5 (6) Connect the name of the angle to the correct angle with a line.

acute

obtuse

right



R W

- G 6 (7) Draw pictures to show parallel and perpendicular lines. Label them.

R W

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

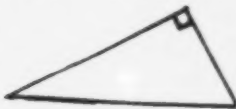
Teacher _____

Student _____

GRADE 7

TEST 1

G 7 (8) Circle one word in each list to describe this triangle:



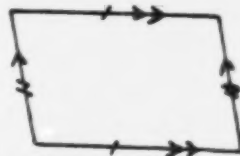
(a) acute, right, obtuse.

R W

(b) isosceles, scalene, equilateral.

R W

G 8 (9) (a) Give the most appropriate name for this quadrilateral.

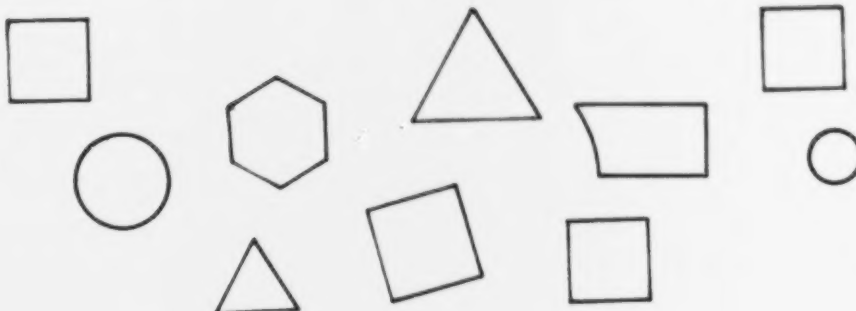


R W

(b) Name a quadrilateral which has more than 2 lines of symmetry.

R W

G9 (10) Mark an "X" on all the shapes that are congruent.



R W

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

Teacher _____

Student _____

GRADE 7

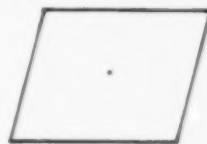
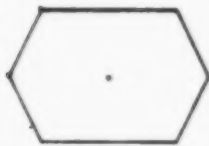
TEST 1

- G 10 Draw a line of symmetry in this equilateral triangle.



R W

- G 11 Circle the shapes that have rotational symmetry about the point shown.



R W

- G 12 (13) Use compasses and a ruler to construct an equilateral triangle with sides of 4 cm.

R W

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

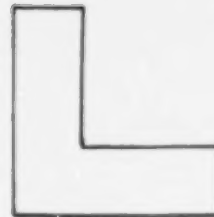
Teacher _____

Student _____

GRADE 7

TEST 1

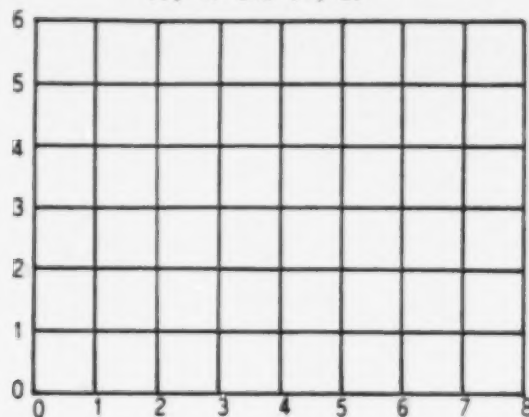
G 14 (15) Draw one line of symmetry in this drawing.



R W

G 15 (16) Mark these points on the grid and draw a line to join them.

(6, 4) and (1, 2)



R W

G 17 (18) Indicate the appropriate term; flip, slide, or turn for moving:

A to B , A to C and A to D



A



B



C



D

R W

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

Teacher _____

Student _____

GRADE 7

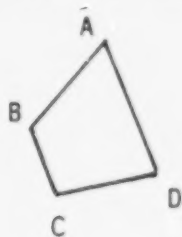
TEST 1

- G 18 (20) Mark a dotted line showing symmetry in this design.
Shade 2 small sections of the design which are congruent.



R W

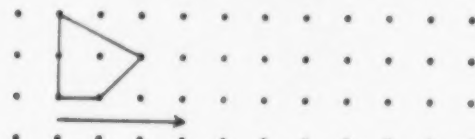
- G 19 (22) Quadrilateral ABCD has been rotated one-half turn.
Label the points on the image which correspond to ABCD.



R W

- G 20 (23) Draw the image of each figure below after it is translated by the given slide arrow.

(a)



R W

(b)



R W

THE PEEL BOARD OF EDUCATION

GEOMETRY EVALUATION

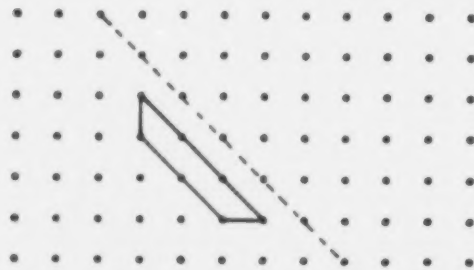
Teacher _____

Student _____

GRADE 7

TEST 1

G 20 (24) Draw the reflection of the figure about the given flip line.



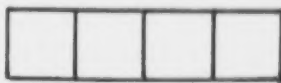
R W

G 20 (25) Draw the image of this shape after making a one-half turn about the point shown.

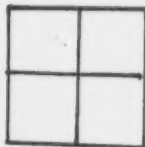


R W

G 21 (26) Mark an "X" on the shapes below which have at least 1 line of symmetry.



R W



Geometry Total R _____

Total W _____

Male or Female _____

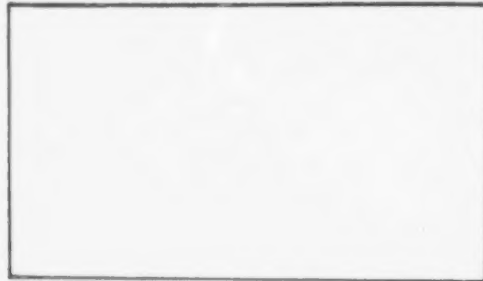
THE PEEL BOARD OF EDUCATION

MEASUREMENT EVALUATION

GRADE 7

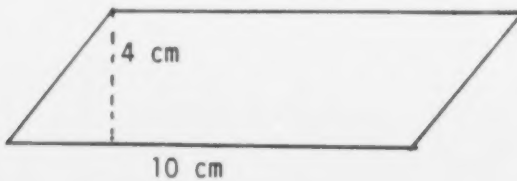
TEST 1

- M 1 Use a ruler to help you measure and calculate the perimeter of this rectangle to the nearest centimetre.



R W

- M 7 Calculate the area of this parallelogram.



R W

Use the correct
unit of measure.

- M 11 Complete the following:

- (a) 3 m = ____ cm (c) 3000 mL = ____ L
(b) 2 km = ____ m (d) 1L = ____ dm³

(a) R W

(b) R W

(c) R W

(d) R W

Measurement

Total R ____

Total W ____

THE PEEL BOARD OF EDUCATION

NUMERICAL METHODS EVALUATION

GRADE 7

TEST 1

N 1 What is the place value of the "6" in the following numerals?

(a) 657 423 _____ (b) 57.016 _____

(a) R W

(b) R W

N 3 (a)
$$\begin{array}{r} 5648 \\ 8269 \\ + 7684 \\ \hline \end{array}$$

(b) $81\,406 - 6818 =$

(a) R W

(b) R W

(c) R W

(c)
$$\begin{array}{r} 486 \\ \times 539 \\ \hline \end{array}$$

(d) $47 \overline{) 57\,998}$

(d) R W

N 5 Calculate:

(a) $4.864 \div 3.2 =$

(a) R W

(b) $37.519 + 516.82 =$

(b) R W

(c) $6.5 \times 3.17 =$

(c) R W

(d) $51.603 - 7.132 =$

(d) R W

Numerical Methods

Total R _____

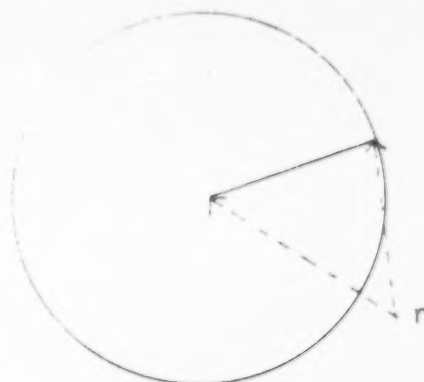
Total W _____

Appendix E
Student Instructional Materials

CONSTRUCTIONS IN GEOMETRY

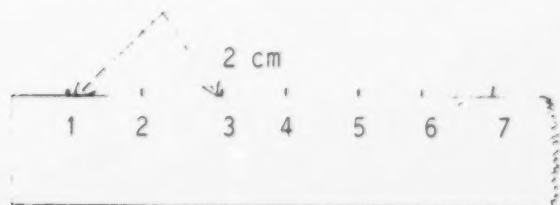
1. Copy a circle:

- (a) Set the compasses at the radius of the given circle:



Mark a point and draw a circle with the radius set.

- (b) Given a radius, set the compasses at the radius given.



Mark a point and draw a circle with the radius set.

2. Copy a line segment:

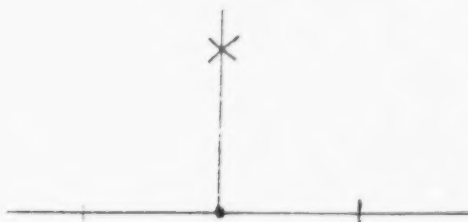


Set your compasses at the length of the given line segment.

Draw a line longer than the given line segment.

Place the compass point on your line and mark a length equal to the compass setting.

3. Construct a perpendicular:



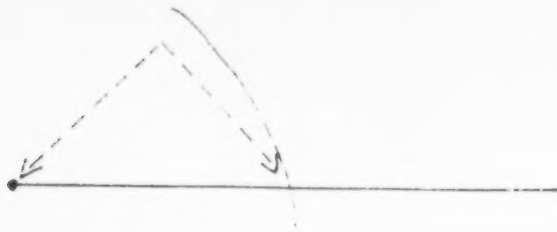
From a point on a given line use the compasses to mark two points equal distances from the chosen point. Using the two marked points as centres describe two arcs that cross above the given line. From the chosen point on the line draw a line passing through the point where the two arcs intersect.

4. Copy an angle:



From the vertex of the angle draw two arcs that intersect both rays of the given angle.

Draw a line. On the line use the compasses to mark an arc the same distance as the arcs on the given angle.



Set the compasses at the distance between the two arcs on the given angle. Use this distance to place an arc above the first arc on your line. Join the end point of your line to the intersection of the two arcs.



5. Copy a triangle:



Draw a line and copy $A B$ of the given triangle (see #2). Set your compasses at distance $A C$ from the original triangle. Use the end of your line segment as a centre and mark an arc above the line segment.



Use B' as a centre with the compasses set at distance $B C$ from the original triangle. Make a second arc above $A'B'$ to intersect the first arc. Join the intersection of the two arcs to A' and B' .



NOTE - a good reference for constructions is "Geometry An Investigative Approach" pages 386 to 403.

Appendix F
Teacher Questionnaires

GEOMETRY STUDY - TEACHER COMMENTS - GROUP A

Thank you for participating in the study of geometry this year. Your comments about your experience are valued and your cooperation in completing the following is appreciated.

- (a) Was the timing and length of the inservice satisfactory?
Yes _____ No _____

COMMENT: _____

- (b) Were the inservice activities appropriate?
Very Useful _____ Fairly Useful _____ Of Little Use _____

COMMENT: _____

- (c) Did the inservice sessions clarify the structures and properties of the geometry?
Very Useful _____ Fairly Useful _____ Of Little Use _____

COMMENT: _____

- (d) Please reflect on the relationship between the inservice experiences and your teaching of geometry.

COMMENT: _____

- (e) Are there any suggestions you would like to make about geometry inservice?

COMMENT: _____

- (f) Were the student instructional materials appropriate for the teaching objectives?

Yes _____ No _____

COMMENT: _____

- (g) Did the instructional materials clarify the structures and properties of geometry for the students?

Very Useful _____ Fairly Useful _____ Of Little Use _____

COMMENT: _____

- (h) Please reflect on the relationship between the student instructional materials and your teaching of geometry.

COMMENT: _____

- (i) Are there any suggestions you would like to make about student instructional materials?

COMMENT: _____

- (j) Were the evaluation forms appropriate for the geometry objectives?

Yes _____ No _____

COMMENT: _____

- (k) Are there any suggestions you would like to make about the evaluation materials?

COMMENT: _____

- (l) Please record any comments you wish to make about the geometry study.

NAME _____
You may choose to identify yourself

GEOMETRY STUDY - TEACHER COMMENTS - GROUP B

Thank you for participating in the study of geometry this year. Your comments about your experience are valued and your cooperation in completing the following is appreciated.

Please return the completed form by April 30, to:

JOAN THOMSON
PROGRAM DEPARTMENT
MATHEMATICS
H.J.A. BROWN EDUCATION CENTRE

- (a) Were the student instructional materials appropriate for the teaching objectives? Yes _____ No _____

COMMENT: _____

- (b) Did the instructional materials clarify the structures and properties of geometry for the students?
Very Useful _____ Fairly Useful _____ Of Little Use _____

COMMENT: _____

- (c) Were the instructional materials as delivered easily implemented?
Very Useful _____ Fairly Useful _____ Of Little Use _____

COMMENTS: _____

- (d) Please reflect on the relationships between the student instructional materials and your teaching of geometry.

COMMENTS: _____

- (e) Are there any suggestions you would like to make about student instructional materials?

COMMENT: _____

- (f) In your opinion would inservice related to geometry assist you?
Yes _____ No _____

COMMENT: _____

- (g) Were the evaluation forms appropriate for the geometry objectives?
Yes _____ No _____

COMMENT: _____

- (h) Are there any suggestions you would like to make about the evaluation materials?

COMMENT: _____

- (i) Please record any comments you wish to make about the geometry study.

NAME _____

You may choose to identify yourself

Appendix G

Canadian Achievement Test Results Report

CLASS RECORD SHEET

Sorted Alphabetically

1981-82

GRADE

SCHOOL

EARNCLIFFE

District Peel Board

Test Date 17-04-82

Raw Date 18-04-82

CANADIAN TEST CENTRE



MCGRAW-HILL RYERSON LIMITED

NAME		Test Level	MATHEMATICS		Total	
			Comp	C & A		
KIYO KAMATA	17	RS	31	***	***	RS
		SS	524	***	***	SS
		NP	63	***	***	NP
		NS	6	***	***	NS
DANIELLE MCCOZ	17	RS	31	31	62	RS
		SS	524	536	530	SS
		NP	63	65	66	NP
		NS	6	6	6	NS
HARMAN MUNDAIR	17	RS	32	29	61	RS
		SS	532	519	526	SS
		NP	68	56	64	NP
		NS	6	5	6	NS
CHRISTIN PARSONS	17	RS	23	26	49	RS
		SS	472	495	476	SS
		NP	30	43	35	NP
		NS	4	5	4	NS
KRISTI PEARCE	17	RS	33	33	66	RS
		SS	540	551	549	SS
		NP	73	72	76	NP
		NS	6	6	6	NS
CARALYN RUBLI	17	RS	31	28	59	RS
		SS	524	512	516	SS
		NP	63	52	58	NP
		NS	6	5	5	NS
CATHERIN SHELTON	17	RS	14	32	46	RS
		SS	415	543	464	SS
		NP	7	68	29	NP
		NS	2	6	4	NS
JEFF SMEETS	17	RS	26	26	52	RS
		SS	490	495	488	SS
		NP	41	43	42	NP
		NS	5	5	5	NS
HALEY WIDEMAN	17	RS	33	29	62	RS
		SS	540	519	530	SS
		NP	73	56	66	NP
		NS	6	5	6	NS
CLASS MEANS		CL	19	17	17	CL
		SS	500	536	513	SS
		NP	46	64	56	NP
		NS	5	6	5	NS

KEY: *** = No score available
RS = Raw Score

CL = # of students who attempted the sub-test

SS = Scale Score

NP = National Percentile

NS = National Stanine

END

U.S. Dept. of Education

Office of Education
Research and
Improvement (OERI)

ERIC

Date Filmed

March 29, 1991